Price indexes linkage.

Ivashchenko Sergey.

Saint-Petersburg State University, Saint-Petersburg, Russia.

JEL classification: E31, E52

Keywords: cointegration; Unit root; DSGE models; sectoral disaggregation;

Abstract

This paper investigates price indexes linkage. Statistical analysis has shown that price indexes linkage is weak (There isn't cointegration between price indexes. Correlation between inflation measures isn't high). Introduction of independent technology development into simple two-sectors DSGE model leads to absence of cointegration between sectors prices.
Integrally, we show that usage of price indexes set should be advantaged. This result reinforces conclusions of other authors about advantage of usage of price indexes for particular problems (monetary policy and forecasting).

Introduction

There are a lot of articles about inflation and monetary policy. But most of them regard price level and inflation as single dimension variable. Usually consumer price index is used as price level.

There are some articles which use few price indexes. Such articles talk that monetary policy could be applied without additional information about sectors dynamic [6]. But it is less effective [2, 4]. Additional price indexes give us additional information from empiric articles point of view. It improves properties of forecasts. [5].

This data put a question why additional price indexes improve situation. Moreover it happens even if wide price indexes are used. There are two possible reasons for such phenomena. First of them is low correlation of sectors inflation. Second reason is absence of cointegration between sectors price indexes.

We start from testing of cointegration between wide price indexes. After that we discuss dependence between inflation measures. Data from Russia and USA would be used for this analysis.

We create dynamic stochastic general equilibrium (DSGE) model with two sectors. This model should explain results of statistical analysis. The model would be estimated for USA. Estimated model would be simulated (linear and quadratic approximation of solution). It shows consequences of technology development independence. At the end we compare results of real and simulated data statistical analysis.

Statistical analysis

There are a lot of dependence forms. But only pairwise dependence would be analyzed within this article.

Engle-Granger technique is used for cointegration testing [7]. Cointegrating vector is searched for price indexes logarithms. It is expected that constant isn't equal to zero at cointegrating vector. So, trend is included at estimated relationship according to Engle-Granger technique.

Cointegration tests were made twice. First time coefficient with price index logarithm was equal to one. It means that stationarity of price indexes ratio was tested. Second time coefficient with price index logarithm was flexible.

Eleven wide price indexes were used for USA. Monthly seasonal adjusted data was used from 1980 till 2007. Results of tests presented at appendix A1. It's easy to find that almost all price indexes combinations aren't cointegrated. Only P1 and P4 are cointegrated according to ADF

test (coefficient equal to 1). And only P1 and P7 are cointegrated according when coefficient is flexible.
R2 of regression between inflation measures was used as indicator of inflation dependence. It could be noted that R2 equal to squared correlation of inflation. It could be found that many combinations explain only small part of variation. It happens despite of its significance. One measure of inflation could explain less then half of variation for more then 70% of combinations. For example, R2 for Consumer Prices, All items less food and energy, SA and Producer Prices, Finished goods less foods and energy, SA is equal to 0,1651.
Thus, price indexes have weak linkage according to statistical data from USA.

The same analysis was made for Russian data. Monthly data from 1999 till 2007 for 5 price indexes was used. Results are presented at appendix A2. There isn't any cointegrated combination of price indexes when coefficient is flexible. And only P1 and P3 are cointegrated according to ADF test (coefficient equal to 1). R2 is much less then 0.5 for all combinations except P1, P2. It is combination of CPI and CPI, food. Moreover, part of combinations isn't significant at 5%.
Thus, Russian price indexes have even weaker linkage then at the USA.

In other words, group factors for prices much les important than specific factors connected to particular sector. Technology development is one of noticeable specific factors. DSGE model with independent by sectors technology development would be presented in the following part.

DSGE model

There are three classes of agents in the DSGE model. First of them are householders. Second are firms which operate on monopolistic competition market. They are divided into two sectors. And the third is a central bank. Each of agent's type sets the group of variables according to its goal or rule.

Monopolistic competition market consists of noncountable set of firms. They are indexed from 0 till 1. Demand for firm's goods is determined by demand for goods basket which forms as following [1]:

$$C_t = \left(\int_0^1 C_{j,t}^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)} \quad (1)$$

, where $C_{j,t}$ – consumption volume of good j at period t.

Solution of expenditure minimization problem determines the pattern of basket:

$$\int_0^1 C_{j,t} P_{j,t} dj \to \min_{C_{j,t}: C_t = \left(\int_0^1 C_{j,t}^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)}} \quad (2)$$

Solution of that problem would be Lagrange coefficient P_t and volume of consumption:

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} C_t \quad (3)$$

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dj \right)^{1/(1-\theta)} \quad (4)$$

Each firms solve problem of the expected capitalized profit maximization.

$$E_0\left(\sum_{t=0}^{\infty}\left(\prod_{\tau=1}^{t}R_{B,\tau}\right)^{-1}\left(P_{j,t}Y_{j,t}-W_tN_{j,t}-R_{r,t}K_{j,t-1}P_t\right)\right)\rightarrow \max \tag{5}$$

, where $Y_{j,t}$ – output of firm j at period t, $N_{j,t}$ – labor demand of firm j at period t, $K_{j,t-1}$ – capital volume used by firm j at period t. Production function of firms is Cobb-Douglas:

$$Y_{j,t}=Z^i_{Y,t}\left(N_{j,t}\right)^{\alpha}\left(K_{j,t}\right)^{1-\alpha} \tag{6}$$

, where $Z^i_{Y,t}$ – reflects technological development at sector i. Sector №1 consist of firms which number is less then φ. Sector №2 consist of al other firms. Demand for firm's goods is following:

$$Y_{j,t}=\left(\frac{P_{j,t}}{P_t}\right)^{-\theta}\left(C_t+I_t\right)=\left(\frac{P_{j,t}}{P_t}\right)^{-\theta}Y_t \tag{7}$$

, where Y_t – aggregate supply, which consist of consumption and investments. So, firm should determine value of following variables: $Y_{j,t}$, $P_{j,t}$, $N_{j,t}$ и $K_{j,t-1}$ at each period t. Maximization of objective function with 2 restrictions leads to such decision.

It could be proved that firm's solution is following:

$$W_tN_{j,t}=P_{j,t}Y_{j,t}\frac{\alpha(\theta-1)}{\theta} \tag{8}$$

$$R_{r,t}K_{j,t}P_t=P_{j,t}Y_{j,t}\frac{(1-\alpha)(\theta-1)}{\theta} \tag{9}$$

Problem of each firm within sector are the same. Therefore, solutions of those problems would be the same. Thus, following equations could be written:

$$K_{t-1}=\int_0^1 K_{j,t}dj=\int_0^{\varphi}K_{j,t}dj+\int_{\varphi}^1 K_{j,t}dj=\varphi K_{1,t}+(1-\varphi)K_{2,t} \tag{10}$$

$$N_t=\int_0^1 N_{j,t}dj=\int_0^{\varphi}N_{j,t}dj+\int_{\varphi}^1 N_{j,t}dj=\varphi N_{1,t}+(1-\varphi)N_{2,t} \tag{11}$$

$$P_t=\left(\int_0^1 P_{j,t}^{1-\theta}dj\right)^{1/(1-\theta)}=\left(\varphi P_{1,t}^{1-\theta}+(1-\varphi)P_{2,t}^{1-\theta}\right)^{1/(1-\theta)} \tag{12}$$

$$\begin{aligned}Y_t=C_t+I_t&=\left(\int_0^1 C_{j,t}^{(\theta-1)/\theta}dj\right)^{\theta/(\theta-1)}+\left(\int_0^1 I_{j,t}^{(\theta-1)/\theta}dj\right)^{\theta/(\theta-1)}=\\&=\left(C_t+I_t\right)\left(\left(\varphi\left(\frac{P_{1,t}}{P_t}\right)^{1-\theta}+(1-\varphi)\left(\frac{P_{2,t}}{P_t}\right)^{1-\theta}\right)^{\theta/(\theta-1)}\right)=\left(\varphi\left(Y_{1,t}\right)^{(\theta-1)/\theta}+(1-\varphi)\left(Y_{2,t}\right)^{(\theta-1)/\theta}\right)^{\theta/(\theta-1)}\end{aligned} \tag{13}$$

It should be noticed that profit at each sector is positive. Following equations should be noticed too:

$$\int_0^1 P_{j,t}Y_{j,t}dj=\left(\varphi P_{1,t}\left(\frac{P_{1,t}}{P_t}\right)^{-\theta}+(1-\varphi)P_{2,t}\left(\frac{P_{2,t}}{P_t}\right)^{-\theta}\right)Y_t=P_tY_t$$

$$\Pi_t=\int_0^1\left(P_{j,t}Y_{j,t}-W_tN_{j,t}-R_{r,t}K_{j,t-1}P_t\right)dj=\int_0^1\left(P_{j,t}Y_{j,t}\left(1-\frac{\theta-1}{\theta}\right)\right)dj=P_tY_t\frac{1}{\theta} \tag{14}$$

Householders are solving problem of its expected capitalized utility maximization with restrictions:

$$E_0\left(\sum_{t=0}^{\infty} Z_{\beta,t} U_t\left(C_t;N_t;\frac{M_t}{P_t}\right)\right) \to \max \quad (15)$$

$$U_t\left(C_t;N_t;\frac{M_t}{P_t}\right) = \log(C_t) - Z_{N,t}\log(N_t) + Z_{M,t}\log\left(\frac{M_t}{P_t}\right) \quad (16)$$

, where N_t – labor supply at period t, M_t – money supply at period t, P_t – price of goods basket at period t, C_t – consumption goods basket at period t.

Process $Z_{N,t}$ represents the element of demographical evolution, which changes the amount of labor being at the disposal of householders. Process $Z_{\beta,t}$ is a stochastic discount factor. Money at the utility function is a common way for DSGE models.

Budget restriction of householders is following:

$$P_t C_t + P_t I_t + M_t + B_t = W_t N_t + M_{t-1} + R_{B,t-1} B_{t-1} + R_{r,t} K_{t-1} P_t + \Pi_t \quad (17)$$

, where I_t – investments volume at period t (determines by the same way as consumption basket), B_t – value of bonds bought at period t, W_t – nominal wage at period t, $R_{B,t}$ – return of bonds bought at period t, $R_{r,t}$ – return of real capital at period t, K_t – volume of capital at the end of period t. The volume of capital depends on investment as following:

$$K_t = (1-\delta)K_{t-1} + I_t \quad (18)$$

Pattern of consumption is determined. So, householders should determine the value of following variables: C_t, I_t, M_t, B_t, N_t и K_t at each period t. Maximization of the utility function with 2 restrictions leads to such decision.

It could be proved that householder's solution is following:

$$Z_{N,t} = \frac{W_t N_t}{P_t C_t} \quad (19)$$

$$\frac{Z_{M,t}}{M_t} = \frac{1}{P_t C_t} - E_t\left(\frac{Z_{\beta,t+1}}{Z_{\beta,t}}\frac{1}{P_{t+1}C_{t+1}}\right) \quad (20)$$

$$\frac{1}{P_t C_t} = R_{B,t} E_t\left(\frac{Z_{\beta,t+1}}{Z_{\beta,t}}\frac{1}{P_{t+1}C_{t+1}}\right) \quad (21)$$

$$\frac{1}{C_t} = E_t\left(\frac{Z_{\beta,t+1}}{Z_{\beta,t}}\frac{1}{C_{t+1}}\left(1-\delta+R_{r,t+1}\right)\right) \quad (22)$$

Following rule describes the policy of central bank:

$$\gamma_R\left(\ln(R_{B,t}) - \eta\ln(R_{B,t-1})\right) + \gamma_M \ln\left(\frac{M_t}{M_{t-1}}\right) = \gamma_P \ln\left(\frac{P_t}{P_{t-1}}\right) + \gamma_Y \ln\left(\frac{Y_t}{Y_{t-1}}\right) + \gamma_1 + z_{R,t} \quad (23)$$

$$\gamma_P = \sqrt{1-\gamma_R^2-\gamma_M^2-\gamma_1^2-\gamma_Y^2}$$

This rule is generalization of the most popular rules. For example, if γM equal to zero then generalized rule equal to Taylor rule (standard coefficient could be calculated through dividing by γR) [1]. If γR, γP and γY equal to zero then Fisher constant money growth rule is taken. McCallum rule could be taken if γR equal to zero and γP equal to γY [8]. There is small difference from rule at [8]. Constant is used instead of average velocity.

γP depend on other coefficient. It provides chose between equivalent rules. It normalize vector (γR;γM;γP;γY;γ1)T. γP is chosen for simplification of interpretation.

Estimation and simulation.

Estimation of linearized model is common way of DSGE models estimation. However, some simplification could be done before linearization. Simplified equations and some details of linearization represented at appendix B1.

Linearization was made around steady-state point. It requires revealing of trends (For unit root variables revealing of drifts). There are 4 additional free coefficients which determine steady-state point. It happens due to existence of cointegration. These additional coefficients became parameters.
Solution of linear model with rational expectations is VAR model according to Blanhard-Kahn. Our model implies that some variables are unit root. It requires transformation of solution to VEC form.
Estimation was made by Bayesian method. List of statistical data and results of estimation presented at appendix B2. Measurement errors were added, since number of observed variables was higher then number of shocks [9]. Combination of transformation to VEC form and measurement errors require changes at Kalman filter. Modified filter presented at appendix C1.
Comments about a prior distribution of parameters:

- Prior distributions of parameters are independent. It's done for simplification of calculation. It fit with usual technology.
- Capital depreciation coefficient (δ) equal to 0,025 at many articles [3, 9]. The same value was used as prior expectation.
- Deterministic discount factor is used usually. It's represented as βt. Usually, β is close to 0,99 [6]. However, smaller values could be find (0,95 at [9], 0,901 at [1]). So, we used beta distribution as prior for q0β. It's mean and standard deviation was chosen according to β values at literature.
- Coefficient α of production function is about 0,64-0,7 [3, 4]. We used the same value as mean of prior distribution.
- There are different values of θ at literature. It's value equal to 6 at [3], and 10 at [6]. Expectation of prior distribution is equal to 8 at this article. Standard deviation would be high (equal to 5).
- Prior distribution mean of φ is equal to 0,5.
- Value of η could be found at Taylor rule. Coefficient with lag of interest rate is usually close to 0,8 [1, 4]. Prior distribution of other monetary policy rule coefficient would be fount through maximum likelihood estimation (see later).
- Prior distribution mean of TN is equal to mean of employment growth(mean(dlog(Nt)).
- Prior distribution mean of q0,M is based on B1.23. It is equal to mean(log(Mt(RB,t-1))-log(PtCtRB,t))
- Prior distribution of average technological growth (q0,Y) would be based on idea that capital output ratio is constant. It means that mean(dlog(Kt))=mean(dlog(Yt))=mean(dlog(ZY,t)+αdlog(Nt)+(1- α)dlog(Kt)).
- Prior distribution mean of monetary policy coefficients, free steady-state coefficients, standard deviations of shocks and measurement errors is based on maximum likelihood estimation (All other parameters was fixed according to its prior means). Standard deviations of this priors are high (for standard deviations it is infinite).

Statistical data of USA was used for estimation of model. It was quarterly data from 1980 till 2007. Full list of statistical series and its transformation represented at appendix B2. Observed variables are C, I, M, N, P, R_B and W.
Properties of prior and posterior parameters distribution represented at appendix B2. It should be noted that part of posterior standard deviations are extremely low. It could be explained by cointegration which makes super-consistent estimator. T-ratio is especially high for gama, q0Y and tN. It could be noted that this parameters determines drift of unit root variables.
There is additional important detail. Posterior mean is close to prior mean. It indicates that prior conform to statistical data (which was used for forming of some prior distributions).
It could be noted that monetary policy rule is far from Taylor rule. Coefficient with inflation is a little bit higher then money growth coefficient. All other coefficients are much lower by absolute

value. Thus, money supply growth has low elasticity by interest rate. But money growth controlling isn't source of antiinflationary actions.

Posterior distribution mean was used for simulation of model. 1000 observations of prices and output ware generated. First and second order approximation was used for simulation. Quadratic approximation was solved according to algorithm [10].
Simulated data was used by the same way as statistical data. It means that R2 of inflation measures regressions represented. Cointegration tests (with fixed and with flexible coefficient) were done. All results represented at appendix B2.

There isn't cointegration between price indexes. However, tests statistics are closer to 5%level for quadratic approximation. R2 are high. This differ simulated data results from statistical. R2 for quadratic approximation are higher.
Thus, independent development of technology explains only part of wake linkage between prices. It's possible that difference at technology or capital and labor heterogeneity could explain low correlation of inflation measurements.

Conclusion

Price indexes linkage was analyzed at this article. Statistical data show that wide price indexes have weak linkage. Correlation of inflation measures isn't high. There isn't cointegration between indexes (Except few combinations of test and price indexes).
DSGE model was created. Monopolistic competition firms were divided into two sectors. Unique difference between them is independent technology development. Parameters were estimated for USA data by Bayesian method.
This assumption provides absence of cointegration between prices for first and second order approximation. However, inflation measures correlation was high.
Thus, there is not cointegration between prices both for statistical data and for model with simple assumptions. Hence, substitution of models aggregate variables to data aggregated by authorities could lead to wrong inference in terms of cointegration. Additional price indexes should be used to avoid such problems. Low correlation of inflations expands these conclusions for models which doesn't works with unit root variables.

Integrally, we show that usage of price indexes set should be advantaged. This result reinforces conclusions about advantage of usage of price indexes for particular problems (monetary policy and forecasting).

Literature

1. Amisano, G., and O. Tristani: “Euro Area Inflation Persistence in an Estimated Nonlinear DSGE Model” 2007, Discussion Paper 6373, C.E.P.R.
2. Carlstrom, C., Fuerst, T. and Ghironi, F.: “Does it matter (for equilibrium determinacy) what price index the central bank targets?” Journal of Economic Theory, 2006, 128, 214.231.
3. Dib, A.: “An Estimated Canadian DSGE Model with Nominal and Real Rigidities” 2001, Bank of Canada Working Paper 2001-26.
4. Erceg, Christopher, and Andrew Levin: “Optimal Monetary Policy with Durable and Nondurable Goods.” 2002 FRB International Finance Discussion Paper 748 and ECB Working Paper 179.
5. Antoni Espasa and Rebeca Albacete: “Econometric modelling for short-term inflation forecasting in the euro area” Journal of Forecasting, 2007, vol. 26, issue 5, pages 303-316
6. Huang, Kevin X.D., and Zheng Liu. “Inflation Targeting: What Inflation Rate to Target?” 2005, Journal of Monetary Economics. 52: 1435-1462.
7. MacKinnon, J.G.: “Critical Values for Cointegration Tests,” 1991, Chapter 13 in Long-run Economic Relationships: Readings in Cointegration, edited by R.F.Engle and C.W.J. Granger, Oxford University Press.
8. Razzak, W, A: “Is the Taylor rule really different from the McCallum rule?” 2001, Reserve Bank of New Zealand Discussion Paper Series DP2001/07
9. Ruge-Murcia, Francisco (2003) “Methods to estimate dynamic stochastic general equilibrium models,” University of Montreal and CIREQ working paper n17.
10. Stephanie Schmitt-Grohe and Martin Uribe: “Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function” 2001, CEPR Working Paper No. 2963, September.

Appendix A1

Table A1.1 List of series

Denotation of series	Name of series
P1	Consumer Prices, All items, SA
P2	Consumer Prices, All items less food and energy, SA
P3	Consumer Prices, All items less energy, SA
P4	Consumer Prices, All items less food, SA
P5	Consumer Prices, All items less food, shelter & energy, SA
P6	Producer Prices, Finished goods total, SA
P7	Producer Prices, Finished goods less foods and energy, SA
P8	Price Index, PCE, Overall, Total, SA
P9	Price Index, PCE, Durable Goods, Overall, Total, SA
P10	Price Index, PCE, Nondurable Goods, Overall, Total, SA
P11	Price Index, PCE, Services, Overall, Total, SA

Table A1.2 Coefficient R^2(regression of the price index logarithms first difference (inflation measures). Name of dependent variable presented at first column)

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
P1	1,0000	0,4318	0,4612	0,9423	0,3423	0,3931	0,1234	0,7169	0,1904	0,6408	0,2045
P2	0,4318	1,0000	0,8842	0,4237	0,6532	0,0264	0,1651	0,3181	0,3615	0,0503	0,3486
P3	0,4612	0,8842	1,0000	0,4027	0,6205	0,0418	0,1570	0,3691	0,3653	0,0795	0,3482
P4	0,9423	0,4237	0,4027	1,0000	0,3182	0,3628	0,1080	0,6901	0,1735	0,6205	0,2002
P5	0,3423	0,6532	0,6205	0,3182	1,0000	0,0441	0,2510	0,3977	0,4874	0,0774	0,3902
P6	0,3931	0,0264	0,0418	0,3628	0,0441	1,0000	0,2332	0,3467	0,0355	0,4772	0,0368
P7	0,1234	0,1651	0,1570	0,1080	0,2510	0,2332	1,0000	0,1799	0,1566	0,0400	0,1873
P8	0,7169	0,3181	0,3691	0,6901	0,3977	0,3467	0,1799	1,0000	0,3273	0,5927	0,5325
P9	0,1904	0,3615	0,3653	0,1735	0,4874	0,0355	0,1566	0,3273	1,0000	0,0275	0,3045
P10	0,6408	0,0503	0,0795	0,6205	0,0774	0,4772	0,0400	0,5927	0,0275	1,0000	0,0234
P11	0,2045	0,3486	0,3482	0,2002	0,3902	0,0368	0,1873	0,5325	0,3045	0,0234	1,0000

Table A1.3 ADF-test statistics for differences of price indexes (trend is included. Number of augments and 5% critical value are presented at the brackets)

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
P1	NA	-0,3278 (12;-3.4253)	-0,1358 (12;-3.4253)	-3,5798 (1;-3.4249)	-0,2707 (11;-3.4252)	0,3873 (0;-3.4248)	-1,5484 (6;-3.4250)	-1,9379 (1;-3.4249)	0,2371 (3;-3.4249)	-1,0612 (9;-3.4252)	-2,7929 (11;-3.4252)
P2	-0,3278 (12;-3.4253)	NA	-1,8709 (1;-3.4249)	-0,4945 (11;-3.4252)	-0,5584 (2;-3.4249)	0,7342 (5;-3.4250)	-0,6355 (4;-3.4250)	0,5119 (4;-3.4250)	-0,5485 (3;-3.4249)	-0,8485 (11;-3.4252)	-0,7462 (3;-3.4249)
P3	-0,1358 (12;-3.4253)	-1,8709 (1;-3.4249)	NA	-0,5946 (10;-3.4252)	-0,1893 (3;-3.4249)	0,7796 (5;-3.4250)	-1,6647 (4;-3.4250)	0,2973 (3;-3.4249)	-0,5457 (6;-3.4250)	-0,7242 (11;-3.4252)	-1,9874 (9;-3.4252)
P4	-3,5798 (1;-3.4249)	-0,4945 (11;-3.4252)	-0,5946 (10;-3.4252)	NA	0,3351 (10;-3.4252)	0,2132 (0;-3.4248)	-2,1716 (4;-3.4250)	-2,4367 (1;-3.4249)	0,2893 (2;-3.4249)	-1,5673 (10;-3.4252)	-2,5221 (11;-3.4252)
P5	-0,2707 (11;-3.4252)	-0,5584 (2;-3.4249)	-0,1893 (3;-3.4249)	0,3351 (10;-3.4252)	NA	0,9667 (4;-3.4250)	0,5074 (1;-3.4249)	1,62 (4;-3.4250)	-0,9991 (5;-3.4250)	-0,6005 (11;-3.4252)	1,3177 (0;-3.4248)
P6	0,3873 (0;-3.4248)	0,7342 (5;-3.4250)	0,7796 (5;-3.4250)	0,2132 (0;-3.4248)	0,9667 (4;-3.4250)	NA	-0,4384 (11;-3.4252)	0,2853 (0;-3.4248)	0,8827 (3;-3.4249)	-1,6733 (4;-3.4250)	-0,5088 (3;-3.4249)
P7	-1,5484 (6;-3.4250)	-0,6355 (4;-3.4250)	-1,6647 (4;-3.4250)	-2,1716 (4;-3.4250)	0,5074 (1;-3.4249)	-0,4384 (11;-3.4252)	NA	-2,251 (2;-3.4249)	0,1456 (3;-3.4249)	-1,3467 (11;-3.4252)	-2,1172 (1;-3.4249)
P8	-1,9379 (1;-3.4249)	0,5119 (4;-3.4250)	0,2973 (3;-3.4249)	-2,4367 (1;-3.4249)	1,62 (4;-3.4250)	0,2853 (0;-3.4248)	-2,251 (2;-3.4249)	NA	0,6974 (1;-3.4249)	-1,6041 (11;-3.4252)	-3,0345 (9;-3.4252)
P9	0,2371 (3;-3.4249)	-0,5485 (3;-3.4249)	-0,5457 (6;-3.4250)	0,2893 (2;-3.4249)	-0,9991 (5;-3.4250)	0,8827 (3;-3.4249)	0,1456 (3;-3.4249)	0,6974 (1;-3.4249)	NA	-0,4789 (11;-3.4252)	0,6715 (1;-3.4249)
P10	-1,0612 (9;-3.4252)	-0,8485 (11;-3.4252)	-0,7242 (11;-3.4252)	-1,5673 (10;-3.4252)	-0,6005 (11;-3.4252)	-1,6733 (4;-3.4250)	-1,3467 (11;-3.4252)	-1,6041 (11;-3.4252)	-0,4789 (11;-3.4252)	NA	-2,2134 (11;-3.4252)
P11	-2,7929 (11;-3.4252)	-0,7462 (3;-3.4249)	-1,9874 (9;-3.4252)	-2,5221 (11;-3.4252)	1,3177 (0;-3.4248)	-0,5088 (3;-3.4249)	-2,1172 (1;-3.4249)	-3,0345 (9;-3.4252)	0,6715 (1;-3.4249)	-2,2134 (11;-3.4252)	NA

Table A1.4 Engle-Granger test statistics for cointegration between logarithms of price indexes (trend is included. Number of augments and 5% critical value are presented at the brackets)

	Independent variable at the cointegrating equation										
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
P1	NA	-2,2108 (12;-3,806)	-3,1939 (11;-3,8059)	-2,6597 (1;-3,8049)	-2,9687 (11;-3,8059)	-0,6662 (1;-3,8049)	-4,4225 (9;-3,8057)	-3,5506 (1;-3,8049)	-3,317 (11;-3,8059)	-1,1094 (10;-3,8058)	-3,6567 (11;-3,8059)
P2	-1,7342 (12;-3,806	NA	-2,202 (1;-3,8049)	-2,5012 (11;-3,8059)	-3,6989 (13;-3,8061)	-0,7381 (8;-3,8056)	-1,5146 (4;-3,8052)	-0,635 (9;-3,8057)	-2,9391 (3;-3,8051)	-1,1844 (11;-3,8059)	-1,0318 (2;-3,805)
P3	-2,6378 (11;-3,805	-2,2532 (1;-3,8049)	NA	-2,2847 (10;-3,8058)	-2,9771 (9;-3,8057)	-0,558 (7;-3,8055)	-1,8597 (4;-3,8052)	-0,3497 (3;-3,8051)	-3,332 (9;-3,8057)	-1,076 (11;-3,8059)	-1,5705 (9;-3,8057)
P4	-2,7245 (1;-3,8049	-3,1208 (11;-3,8059)	-2,9731 (10;-3,8058)	NA	-2,9807 (11;-3,8059)	-0,7911 (1;-3,8049)	-3,6279 (4;-3,8052)	-3,5962 (1;-3,8049)	-3,3634 (11;-3,8059)	-1,4887 (11;-3,8059)	-3,5921 (11;-3,8059)
P5	-2,3622 (11;-3,805	-3,5048 (13;-3,8061)	-2,8629 (9;-3,8057)	-2,2429 (11;-3,8059)	NA	-0,468 (7;-3,8055)	-1,2656 (3;-3,8051)	-0,2651 (10;-3,8058)	-3,2539 (6;-3,8054)	-1,1144 (11;-3,8059)	-0,2384 (2;-3,805)
P6	-0,9606 (3;-3,8051	-1,6426 (7;-3,8055)	-1,5927 (7;-3,8055)	-0,9623 (3;-3,8051)	-1,7373 (7;-3,8055)	NA	-1,4918 (7;-3,8055)	-1,3448 (3;-3,8051)	-1,8516 (7;-3,8055)	-2,3022 (0;-3,8048)	-1,6525 (7;-3,8055)
P7	-3,9278 (9;-3,8057	-1,8904 (4;-3,8052)	-2,1511 (4;-3,8052)	-3,2879 (4;-3,8052)	-1,8914 (3;-3,8051)	-0,6277 (10;-3,8058)	NA	-2,5872 (2;-3,805)	-2,6706 (3;-3,8051)	-1,1292 (10;-3,8058)	-1,7388 (1;-3,8049)
P8	-3,1711 (1;-3,8049	-1,0465 (10;-3,8058)	-0,8144 (4;-3,8052)	-3,1666 (1;-3,8049)	-0,6787 (10;-3,8058)	-0,3699 (3;-3,8051)	-2,6735 (2;-3,805)	NA	-1,5854 (10;-3,8058)	-1,7335 (11;-3,8059)	-2,5831 (10;-3,8058)
P9	-2,5361 (11;-3,805	-2,4664 (3;-3,8051)	-3,0136 (9;-3,8057)	-2,457 (11;-3,8059)	-3,0149 (6;-3,8054)	-0,372 (10;-3,8058)	-1,8249 (3;-3,8051)	-0,8504 (9;-3,8057)	NA	-1,0537 (11;-3,8059)	-1,0901 (2;-3,805)
P10	-1,9429 (11;-3,805	-2,3712 (11;-3,8059)	-2,3389 (11;-3,8059)	-2,029 (11;-3,8059)	-2,3404 (11;-3,8059)	-1,3681 (6;-3,8054)	-2,3522 (11;-3,8059)	-2,5303 (11;-3,8059)	-2,4183 (11;-3,8059)	NA	-2,7745 (11;-3,8059)
P11	-3,4368 (11;-3,805	-1,5743 (2;-3,805)	-1,9291 (9;-3,8057)	-3,2764 (11;-3,8059)	-1,6218 (2;-3,805)	-0,9974 (3;-3,8051)	-1,9377 (1;-3,8049)	-2,6679 (10;-3,8058)	-2,4917 (4;-3,8052)	-2,1286 (11;-3,8059)	NA

Appendix A2

Table A2.1 List of series

Denotation of series	Name of series
P1	CPI
P2	CPI, food goods
P3	CPI, nonfood goods
P4	CPI, chargeable services
P5	PPI

Table A2.2 Coefficient R^2(regression of the price index logarithms first difference (inflation measures). Name of dependent variable presented at first column)

	P1	P2	P3	P4	P5
P1	1.0000000	0.7841504	0.3422871	0.4015945	0.0585997
P2	0.7841504	1.0000000	0.1295701	0.0878178	0.0229612
P3	0.3422871	0.1295701	1.0000000	0.0579769	0.2228013
P4	0.4015945	0.0878178	0.0579769	1.0000000	0.0032049
P5	0.0585997	0.0229612	0.2228013	0.0032049	1.0000000

Table A2.3 ADF-test statistics for differences of price indexes (trend is included. Number of augments and 5% critical value are presented at the brackets)

	P1	P2	P3	P4	P5
P1	NA	0.347179 (8, -3.4552)	-4.056655 (7, -3.4548)	-0.695242 (12, -3.4571)	-2.412279 (2, -3.4527)
P2	0.347179 (8, -3.4552)	NA	-3.009082 (5, -3.4539)	-0.102448 (12, -3.4571)	-2.934653 (2, -3.4527)
P3	-4.056655 (7, -3.4548)	-3.009082 (5, -3.4539)	NA	-1.905112 (12, -3.4571)	-1.830264 (2, -3.4527)
P4	-0.695242 (12, -3.4571)	-0.102448 (12, -3.4571)	-1.905112 (12, -3.4571)	NA	-1.340549 (1, -3.4523)
P5	-2.412279 (2, -3.4527)	-2.934653 (2, -3.4527)	-1.830264 (2, -3.4527)	-1.340549 (1, -3.4523)	NA

Table A2.4 A1.4 Engle-Granger test statistics for cointegration between logarithms of price indexes (trend is included. Number of augments and 5% critical value are presented at the brackets)

	Independent variable at the cointegrating equation				
	P1	P2	P3	P4	P5
P1	NA	-1.376651 (7, -3.878717111)	-2.118760 (7, -3.878717111)	-1.808176 (5, -3.876653305)	-1.054727 (5, -3.876653305)
P2	-1.902652 (7, -3.878717111)	NA	-3.487769 (5, -3.876653305)	-1.989609 (7, -3.878717111)	-1.555755 (7, -3.878717111)
P3	-2.684414 (7, -3.878717111)	-3.367162 (4, -3.875651491)	NA	-3.411849 (12, -3.884257268)	-1.127575 (2, -3.873705174)
P4	-1.084778 (7, -3.878717111)	-0.899670 (7, -3.878717111)	-3.130455 (12, -3.884257268)	NA	-2.534251 (12, -3.884257268)
P5	-2.568851 (2, -3.873705174)	-2.828101 (2, -3.873705174)	-2.045889 (2, -3.873705174)	-2.996327 (2, -3.873705174)	NA

Appendix B1

Equations at the initial form:

$$Y_{j,t} = Z^i_{Y,t} Z \left(N_{j,t}\right)^{\alpha} \left(K_{j,t}\right)^{1-\alpha} \tag{B1.1}$$

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} \left(C_t + I_t\right) = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t \tag{B1.2}$$

$$W_t N_{j,t} = P_{j,t} Y_{j,t} \frac{\alpha(\theta - 1)}{\theta} \tag{B1.3}$$

$$R_{r,t} K_{j,t} P_t = P_{j,t} Y_{j,t} \frac{(1-\alpha)(\theta - 1)}{\theta} \tag{B1.4}$$

$$\Pi_t = P_t Y_t \frac{1}{\theta} \tag{B1.5}$$

$$K_{t-1} = \varphi K_{1,t} + (1-\varphi) K_{2,t} \tag{B1.6}$$

$$N_t = \varphi N_{1,t} + (1-\varphi) N_{2,t} \tag{B1.7}$$

$$P_t = \left(\varphi P_{1,t}^{1-\theta} + (1-\varphi) P_{2,t}^{1-\theta}\right)^{1/(1-\theta)} \tag{B1.8}$$

$$Y_t = C_t + I_t = \left(\varphi \left(Y_{1,t}\right)^{(\theta-1)/\theta} + (1-\varphi)\left(Y_{2,t}\right)^{(\theta-1)/\theta}\right)^{\theta/(\theta-1)} \tag{B1.9}$$

$$P_t C_t + P_t I_t + M_t + B_t = W_t N_t + M_{t-1} + R_{B,t-1} B_{t-1} + R_{r,t} K_{t-1} P_t + \Pi_t \tag{B1.10}$$

$$K_t = (1-\delta) K_{t-1} + I_t \tag{B1.11}$$

$$Z_{N,t} = \frac{W_t N_t}{P_t C_t} \tag{B1.12}$$

$$\frac{z_{M,t}}{M_t} = \frac{1}{P_t C_t} - E_t\left(\frac{Z_{\beta,t+1}}{Z_{\beta,t}} \frac{1}{P_{t+1} C_{t+1}}\right) \tag{B1.13}$$

$$\frac{1}{P_t C_t} = R_{B,t} E_t\left(\frac{Z_{\beta,t+1}}{Z_{\beta,t}} \frac{1}{P_{t+1} C_{t+1}}\right) \tag{B1.14}$$

$$\frac{1}{C_t} = E_t\left(\frac{Z_{\beta,t+1}}{Z_{\beta,t}} \frac{1}{C_{t+1}} \left(1 - \delta + R_{r,t+1}\right)\right) \tag{B1.15}$$

$$\gamma_R\left(\ln\left(R_{B,t}\right) - \eta \ln\left(R_{B,t-1}\right)\right) + \gamma_M \ln\left(\frac{M_t}{M_{t-1}}\right) = \gamma_P \ln\left(\frac{P_t}{P_{t-1}}\right) + \gamma_Y \ln\left(\frac{Y_t}{Y_{t-1}}\right) + \gamma_1 + z_{R,t}$$

$$\gamma_P = \sqrt{1 - \gamma_R^2 - \gamma_M^2 - \gamma_1^2 - \gamma_Y^2} \tag{B1.16}$$

$$d\log(Z^i_{Y,t}) = \log(Z^i_{Y,t}) - \log(Z^i_{Y,t-1}) = q_{0,Y} + \varepsilon^i_{Y,t} \tag{B1.17}$$

$$d\log(Z_{N,t}) = \log(Z_{N,t}) - \log(Z_{N,t-1}) = q_{0,N} + \varepsilon_{N,t} \tag{B1.18}$$

$$\log(z_{M,t}) = q_{0,M} + \varepsilon_{M,t} \tag{B1.19}$$

$$d\log(Z_{\beta,t}) = \log(Z_{\beta,t}) - \log(Z_{\beta,t-1}) = q_{0,\beta} + \varepsilon_{\beta,t} \tag{B1.20}$$

$$z_{R,t} = \varepsilon_{R,t} \tag{B1.21}$$

Simplification:

B1.2, B1.3, B1.5 make possible to transform B1.10 into:

$$M_t + B_t = M_{t-1} + R_{B,t-1} B_{t-1} \tag{B1.22}$$

B1.14 make possible to transform B1.13 into:

$z_{M,t}P_tC_tR_{B,t} = M_t(R_{B,t} - 1)$ (B1.23)

$Y_t = \left(\varphi\left(Y_{1,t}\right)^{(\theta-1)/\theta} + (1-\varphi)\left(Y_{2,t}\right)^{(\theta-1)/\theta}\right)^{\theta/(\theta-1)}$ is true because of B1.2 and B1.8. Thus, B1.9 is equal to:

$Y_t = C_t + I_t$ (B1.24)

Transformation of bond volume would be done:

$B_t = b_t M_t$

After that log-linearization is done (For b_t linearization is done for possibility of negative values of B_t). Log-linearization is made around steady-state. But trends (drift for unit root variables) should be eliminated for existence of steady-state. List of variables and drifts presented at table B1.1

Table B1.1 Deterministic trends

List of variables	Trend
M	$\tau_M = \dfrac{(\gamma_P - \gamma_Y)(\tau_N + q_{0,Y}) - \gamma_1 - \gamma_R q_{0,\beta}(1-\eta)}{-\gamma_M - \gamma_R(1-\eta) + \gamma_P}$
N, N_1 ,N_2	τ_N=- $q_{0,N}$
P, P_1, P_2	τ_M - τ_N - $q_{0,Y}$
K, K_1, K_2, Y, Y_1, Y_2, C, I	τ_N + $q_{0,Y}$
W	τ_M - τ_N
Z^1_Y, Z^2_Y	$q_{0,Y}$
Z_N	$q_{0,N}$
Z_β	$q_{0,\beta}$
R_B, R_r, z_R, z_M, b	0

Appendix B2

Table B2.1 List of statistical series

Denotation	Statistical series
X1	Personal Outlays, PCE, Overall, Total, Current Prices, AR, SA, USD'
X2	Personal Outlays, PCE, Overall, Total, Constant Prices, AR, SA, USD'
X3	Investment Account, Gross Private Domestic Investment, Total, Current Prices, AR, SA, USD'
X4	Investment Account, Gross Private Domestic Investment, Total, Constant Prices, AR, SA, USD'
X5	Price Index, Gross Domestic Product, Total, SA, Index'
X6	Personal Income Account, Wage and Salary Disbursements, Total, Current Prices, AR, SA, USD'
X7	Employment, Overall, Total, usually work full time, SA'
X8	Earnings, Average Weekly, Nonfarm payroll, total private, SA, USD'
X9	Treasury Bills, Bid, 3 Month, Yield, Average, USD'
X10	Money supply M1, SA, USD'

Table B2.2 List of models observed variables.

Denotation	transformation of statistical series
M	X10
R_B	(1+X9/100)/4
W	0.25*X6/X7
N	X7
P	X5
I	X4/4
C	X2/4

Table B2.3 Properties of prior and posterior distributions

	Prior distribution			Posterior distribution		
Parameter	Name	mean	Standard deviation	mean	Standard deviation	t-ratio
δ	beta_pdf	2,500E-02	0,015	2,590E-02	2,384E-02	1,087E+00
α	beta_pdf	6,600E-01	0,2	6,659E-01	1,443E-01	4,615E+00
θ	gamma_pdf	8,000E+00	5	8,026E+00	6,358E-01	1,262E+01
φ	beta_pdf	5,000E-01	0,2	5,005E-01	1,724E-02	2,903E+01
η	beta_pdf	8,000E-01	0,15	8,005E-01	1,248E-02	6,416E+01
γ_1	beta_pdf(-1;1)	5,251E-03	0,5	5,251E-03	5,226E-08	1,005E+05
γ_M	beta_pdf(-1;1)	7,024E-01	0,2	7,024E-01	4,335E-07	1,620E+06
γ_R	beta_pdf(-1;1)	2,290E-02	0,5	2,293E-02	4,287E-04	5,348E+01
γ_Y	beta_pdf(-1;1)	8,288E-02	0,5	8,288E-02	3,170E-04	2,614E+02
τ_N	inv_gamma_pdf	3,410E-03	3,00E-03	3,410E-03	2,619E-08	1,302E+05
$q_{0,\beta}$	beta_pdf(-0.2;0)	-5,000E-02	0,03	-5,000E-02	1,049E-04	-4,767E+02
$q_{0,Y}$	beta_pdf	4,741E-03	3,00E-03	4,740E-03	8,937E-09	5,304E+05
$q_{0,M}$	normal_pdf	-9,289E+00	5	-9,291E+00	6,774E-03	-1,372E+03
P_0	normal_pdf	-1,848E+00	5	-1,848E+00	4,500E-04	-4,107E+03
N_0	normal_pdf	-3,639E+01	50	-3,640E+01	6,104E-03	-5,963E+03
$Z^1_{Y,0}$	normal_pdf	-1,396E+00	5	-1,396E+00	1,573E-04	-8,878E+03
$Z^2_{Y,0}$	normal_pdf	-1,256E+00	5	-1,256E+00	8,438E-05	-1,488E+04
Stderr ε_β	inv_gamma_pdf	2,862E-02	inf	2,700E-02	2,741E-02	9,850E-01
stderr ε_M	inv_gamma_pdf	1,471E-02	inf	1,518E-02	1,018E-01	1,491E-01
stderr ε_N	inv_gamma_pdf	2,353E-04	inf	2,353E-04	2,959E-08	7,952E+03
stderr ε_R	inv_gamma_pdf	7,239E-03	inf	7,107E-03	7,507E-03	9,467E-01
Stderr ε^1_Y	inv_gamma_pdf	1,007E-02	inf	9,489E-03	1,795E-02	5,285E-01
Stderr ε^2_Y	inv_gamma_pdf	2,795E-03	inf	2,222E-03	1,214E-02	1,830E-01
Stderr C	inv_gamma_pdf	1,855E-02	inf	6,793E-03	7,144E-01	9,509E-03
Stderr I	inv_gamma_pdf	1,503E-01	inf	1,748E-01	1,527E+00	1,144E-01
Stderr M	inv_gamma_pdf	1,133E-02	inf	1,100E-02	1,683E-03	6,538E+00
Stderr N	inv_gamma_pdf	1,596E-03	inf	2,529E-03	1,297E-02	1,950E-01
Stderr P	inv_gamma_pdf	1,400E-03	inf	1,342E-03	9,128E-03	1,470E-01
Stderr RB	inv_gamma_pdf	1,551E-03	inf	1,435E-03	1,445E-03	9,931E-01
Stderr W	inv_gamma_pdf	8,356E-02	inf	8,442E-02	5,974E-02	1,413E+00

beta_pdf(a;b) – beta distribution which range of values is (a;b) instead of (0;1).

Table B2.3 Coefficient R^2(regression of the price index logarithms first difference. Name of dependent variable presented at first column)

	P1	P2	P	Y1	Y2	Y
Linear approximation						
P1	1.0000000	0.6278039	0.7968020	0.2147422	0.2316780	0.0419937
P2	0.6278039	1.0000000	0.9648768	0.0299912	0.0229400	0.0384014
P	0.7968020	0.9648768	1.0000000	0.0002072	0.0012924	0.0431576
Y1	0.2147422	0.0299912	0.0002072	1.0000000	0.9891309	0.0003596
Y2	0.2316780	0.0229400	0.0012924	0.9891309	1.0000000	0.0151527
Y	0.0419937	0.0384014	0.0431576	0.0003596	0.0151527	1.0000000
Quadratic approximation						
P1	1.0000000	0.9960867	0.9974352	0.0052019	2.014E-05	0.0131409
P2	0.9960867	1.0000000	0.9997426	0.0001183	0.0020033	0.0139497
P	0.9974352	0.9997426	1.0000000	0.0004673	0.0008419	0.0137080
Y1	0.0052019	0.0001183	0.0004673	1.0000000	0.4159352	0.0003490
Y2	2.014E-05	0.0020033	0.0008419	0.4159352	1.0000000	0.0649359
Y	0.0131409	0.0139497	0.0137080	0.0003490	0.0649359	1.0000000

Table A2.3 ADF-test statistics for differences of price indexes (trend is included. Number of augments is presented at the brackets). 5% critical value=-3.4167

	P1	P2	P	Y1	Y2	Y
Linear approximation						
P1		-1.883570(0)	-1.883570(0)	-4.225133(1)	-3.223226(7)	-2.022098(10)
P2	-1.883570(0)		-1.883570(0)	-4.707402(1)	-3.848064(7)	-1.419475(7)
P	-1.883570(0)	-1.883570(0)		-4.561619(1)	-3.765840(7)	-1.408401(6)
Y1	-4.225133(1)	-4.707402(1)	-4.561619(1)		-1.883570(0)	-1.883570(0)
Y2	-3.223226(7)	-3.848064(7)	-3.765840(7)	-1.883570(0)		-1.883570(0)
Y	-2.022098(10)	-1.419475(7)	-1.408401(6)	-1.883570(0)	-1.883570(0)	
Quadratic approximation						
P1		-2.638170(0)	-2.711252(0)	-0.948771(12)	-3.420533(11)	-3.437912(4)
P2	-2.638170(0)		-2.591429(7)	-1.283367(12)	-2.993734(11)	-3.413686(4)
P	-2.711252(0)	-2.591429(7)		-1.219165(12)	-3.144239(11)	-3.700750(2)
Y1	-0.948771(12)	-1.283367(12)	-1.219165(12)		-2.638170(0)	-2.711252(0)
Y2	-3.420533(11)	-2.993734(11)	-3.144239(11)	-2.638170(0)		-2.591429(7)
Y	-3.437912(4)	-3.413686(4)	-3.700750(2)	-2.711252(0)	-2.591429(7)	

Table A2.4 Engle-Granger test statistics for cointegration between logarithms of price indexes (trend is included. Number of augments is presented at the brackets). Name of independent variables at the cointegrating equation presented at first row. 5% critical value=-3,7841

	P1	P2	P	Y1	Y2	Y
Linear approximation						
P1		-2.162836(0)	-2.174527(0)	-3.404653(0)	-2.504833(1)	-1.208932(9)
P2	-2.204886(0)		-2.174527(0)	-3.393331(0)	-2.561784(1)	-1.064200(7)
P	-2.204886(0)	-2.162836(0)		-3.396503(0)	-2.546523(1)	-1.083156(6)
Y1	-2.723019(0)	-2.667072(0)	-2.682603(0)		-2.658697(0)	-2.883203(0)
Y2	-0.867262(0)	-0.911280(0)	-0.900178(0)	-2.038726(0)		-2.883203(0)
Y	0.466655(5)	0.105936(6)	0.079642(6)	-2.038726(0)	-2.658697(0)	
Quadratic approximation						
P1		-2.544462(0)	-2.558921(0)	1.374636(1)	-0.951558(13)	-4.018052(5)
P2	-2.535776(0)		-2.906909(6)	1.356952(1)	-1.033273(13)	-3.924089(4)
P	-2.551940(0)	-2.908384(6)		1.361497(1)	-1.009212(13)	-3.959806(4)
Y1	-2.272376(0)	-2.282035(0)	-2.280192(0)		-2.820885(0)	-2.861624(0)
Y2	-1.497062(11)	-1.496255(11)	-1.496301(11)	-2.018344(2)		-4.070024(7)
Y	-3.817636(10)	-3.854685(10)	-3.850747(10)	-1.933581(0)	-3.895459(6)	

Appendix C1

Initial model:

$$S_t = AS_{t-1} + \varepsilon_t \quad \text{(C1.1)}$$

$$X_t = HS_t + u_t \quad \text{(C1.2)}$$

$$\underset{n\times n}{A} - \underset{n\times n}{I} = \underset{n\times k}{B}\,\underset{k\times n}{U} \quad \text{(C1.3)}$$

Transformation of model:

$$S_t - S_{t-1} = BUS_{t-1} + \varepsilon_t$$

$$US_t = U(S_t - S_{t-1}) + US_{t-1} = (\underset{k\times k}{I} + UB)US_{t-1} + U\varepsilon_t$$

$$F_t = \begin{bmatrix} US_t \\ S_t - S_{t-1} \end{bmatrix} = \begin{bmatrix} \underset{k\times k}{I} + UB & 0 \\ B & 0 \end{bmatrix} F_{t-1} + \begin{bmatrix} U \\ \underset{n\times n}{I} \end{bmatrix} \varepsilon_t = PF_{t-1} + Q\varepsilon_t \quad \text{(C1.4)}$$

$$X_t - X_{t-1} = H(S_t - S_{t-1}) + u_t - u_{t-1} = H_1 F_t + u_t - u_{t-1} \quad \text{(C1.5)}$$

Initialization:

$$Var(F_t^1) = \left(\underset{k\times k}{I} + UB\right) Var(F_t^1) \left(\underset{k\times k}{I} + UB\right)' + UVar(\varepsilon_t)U' \quad \text{(C1.6)}$$

$$vec(Var(F_t^1)) = \left(\left(\underset{k\times k}{I} + UB\right) \otimes \left(\underset{k\times k}{I} + UB\right)\right) vec(Var(F_t^1)) + (U \otimes U)vec(Var(\varepsilon_t))$$

$$\left(I - \left(\underset{k\times k}{I} + UB\right) \otimes \left(\underset{k\times k}{I} + UB\right)\right) vec(Var(F_t^1)) = (U \otimes U)vec(Var(\varepsilon_t))$$

$$rank\left(I - \left(\underset{k\times k}{I} + UB\right) \otimes \left(\underset{k\times k}{I} + UB\right)\right) = rank\left(I - eig\left(\underset{k\times k}{I} + UB\right) \otimes eig\left(\underset{k\times k}{I} + UB\right)\right) =$$

$$= rank\left(I - \left(\underset{k\times k}{I} + eig(UB)\right) \otimes \left(\underset{k\times k}{I} + eig(UB)\right)\right) = rank\left(\left(-eig(UB)\right) \otimes \left(2\underset{k\times k}{I} + eig(UB)\right)\right)$$

$$k = rank(BU) = rank(BUBU) \le rank(UB) \le k$$

$$rank\left(2\underset{k\times k}{I} + eig(UB)\right) = k$$

$$vec(Var(F_t^1)) = \left(I - \left(\underset{k\times k}{I} + UB\right) \otimes \left(\underset{k\times k}{I} + UB\right)\right)^{-1} (U \otimes U)vec(Var(\varepsilon_t)) \quad \text{(C1.7)}$$

$$Var(F_t^2) = BVar(F_t^1)B' + Var(\varepsilon_t) \quad \text{(C1.8)}$$

$$\mathrm{cov}(F_t^1; F_t^2) = \left(\underset{k\times k}{I} + UB\right) Var(F_t^1)B' + UVar(\varepsilon_t) \quad \text{(C1.9)}$$

Update:

$$E_{t-1}(dX_t) = H_1 E_{t-1}(F_t) - E_{t-1}(u_{t-1}) \quad \text{(C1.10)}$$

$$Var_{t-1}(dX_t) = H_1 Var_{t-1}(F_t) H_1' + Var(u_t) + Var_{t-1}(u_{t-1}) \quad \text{(C1.11)}$$

$$E_t(F_t) = E_{t-1}(F_t) + \left(Var_{t-1}(F_t)\right)\left(H_1'\right)\left(Var_{t-1}(dX_t)\right)^{-1}\left(dX_t - E_{t-1}(dX_t)\right) \quad \text{(C1.12)}$$

$$Var_t(F_t) = \left(I - \left(Var_{t-1}(F_t)\right)H_1'\left(Var_{t-1}(dX_t)\right)^{-1} H_1\right) \mathrm{var}_{t-1}(F_t)$$

$$\left(I - \left(Var_{t-1}(F_t)\right)H_1'\left(Var_{t-1}(dX_t)\right)^{-1} H_1\right)' + \quad \text{(C1.13)}$$

$$+\left(\left(Var_{t-1}(F_t)\right)H_1'\left(Var_{t-1}(dX_t)\right)^{-1}\right)\left(Var(u_t) + Var_{t-1}(u_{t-1})\right)\left(\left(Var_{t-1}(F_t)\right)H_1'\left(Var_{t-1}(dX_t)\right)^{-1}\right)'$$

$$E_t(u_t) = \left(Var(u_t)\right)\left(Var_{t-1}(dX_t)\right)^{-1}\left(dX_t - E_{t-1}(dX_t)\right) \quad \text{(C1.14)}$$

$$Var_t(u_t) = \left(I - Var(u_t)\left(Var_{t-1}(dX_t)\right)^{-1}\right)\mathrm{var}(u_t)\left(I - Var(u_t)\left(Var_{t-1}(dX_t)\right)^{-1}\right) +$$
$$+ \left(Var(u_t)\left(Var_{t-1}(dX_t)\right)^{-1}\right)Var_{t-1}(u_{t-1})\left(Var(u_t)\left(Var_{t-1}(dX_t)\right)^{-1}\right)' + \quad \text{(C1.15)}$$
$$+ \left(Var(u_t)\left(Var_{t-1}(dX_t)\right)^{-1} H_1\right)Var_{t-1}(F_t)\left(Var(u_t)\left(Var_{t-1}(dX_t)\right)^{-1} H_1\right)'$$
$$E_t(F_{t+1}) = PE_t(F_t) \quad \text{(C1.16)}$$
$$Var_t(F_{t+1}) = PVar_t(F_t)P' + QVar(\varepsilon_t)Q' \quad \text{(C1.17)}$$