Bayesian Estimation of a DSGE Model and Convergence Dynamics Analysis for Central Europe Transition Economies*

Tareq Sadeq†
EPEE, Université d'Evry Val d'Essonne

May 2008

Abstract

This paper studies a dynamic stochastic general equilibrium model for a small open economy with two types of consumption goods (home produced goods and imported goods) and two types of production goods (home goods and export goods). The objective is to analyze the convergence issues, with respect to the EU-15, of the Central Europe countries: Czech Republic, Hungary, Poland, Slovakia and Slovenia. The model exhibits features of market imperfections and nominal rigidities. Then, the model is estimated using Bayesian methods. The estimation results illustrate some differences from the Euro area results in structural parameters. However, the results exhibit some similarities across countries, notably in some shocks volatilities and high habit formation of consumption. The results illustrate also an important degree of rigidity of imported goods prices, which implies a low pass-through of the exchange rate fluctuations. Finally, we study the Ramsey optimal allocation, in a timeless perspective, of the estimated model for each country in order to analyze the convergence criteria of entrance in the european exchange rate mechanism.

*I would like to thank first my supervisor Ferhat Mihoubi for all the useful comments and the helpful supervision to achieve this paper. I thank also Douglas Laxton, Paolo Pesenti and Dirk Muir for their feedback and suggestions.
†tareq.sadeq@univ-evry.fr

1 Introduction

In May 2004, ten countries of Central and Eastern Europe joined the EU, this revealed a debate about their adoption of the Euro, since they caracterize some different features of Occidental Europe. The EU accession countries were mostly characterized by high inflation in the middle 1990s and high external debt. In order to join the Euro area, these countries have to satisfy some convergeance criteria in order to access to the european exchange rate mechanism (ERM II) according to the Maastricht treaty, then at least two years after their acession to the ERM II they can adopt the Euro. The ERM II accession criteria require from candidate countries to the EMU, to stabilize their inflation rate, output gap, external debt, interest rate and nominal exchange rate variation within 15%, with respect to the initial level at the accession date to the ERM II. In November 2005, Slovakia and the three baltic countries joined the ERM II. In January 2007 Slovenia, Malta and Cyprus could join the Euro area. However, the Czech Republic, Hungary and Poland are still struggling to stabilize their economies and their exchange rates in order to adopt the ERM II.

The difference of convergence degrees of the transition countries is due to different features across countries, such as nominal rigidities and adjustment parameters, thus different policies, but also because of different volatilities of country-specific shocks.

The litetature did not deal enough with the convergence issues of these countries. Laxton and Pesenti (2003) present a dynamic stochastic global economy model calibrated for the Czech Republic. Then they try to find the optimal monetary policy by comparing the welfare under different monetary rules. Natalucci and Ravenna (2003) examine the choice of exchange rate regime in the ten accession countries in a DSGE framework. However, their model can be highly critisized since it does not include any imperfection of the market, which are important in a DSGE model analysis to explain the short-term smoothness of the economy. Even if the transition economies feature more variation than developped countries, the assumption of no imperfection is extreme. Another criticism to the existant studies on transition countries is the use of calibrated models, as I indicated this is not robust since data samples are not large enough to conclude a such calibration. That's why the use of Bayesian methods may give a better idea about the parameters distribution.

The recent literature on the stochastic dynamic general equilibrium models has focused on market imperfections. Many studies have examined the impact of incomplete pass-through of exchange rate movements on international trade prices and the consumer price index, and which monetary policy should be adopted. They are estimated using mostly Bayesian methods, but also GMM or simulated moments method. Nevertheless, most of these models are done for industrialized economies, for example, the Euro area, the USA, Canada,...*etc.* Few DSGE models are developed for emerging and transition countries.

The Balassa-Samuelson effect is usually present in small open emerging economies. This hypothesis is induced by the fact that prices are systemetically different in foreign developped countries. So, there exists a difference in productivity between emerging and developped countries. As a consequence, exports of the emerging countries are constrained by other prices than the home prices, which introduces a difference between prices of goods for home consumption and exports. As a result, this affects the real exchange rate to deviate from the purchasing power parity (PPP), as show Natalucci and Ravenna. For this reason, we segregate in our model non-tradable home goods and tradable goods (exports) sector.

The DSGE models are important for monetary policy estimation and for the determination of the optimal monetary policy, taking into account all the imperfections. The emerging countries exhibit some particular features. As show Laxton and Pesenti (2003), in their model applied to the Czech Republic, the monetary policy is dependant on the degree of exchange rate pass-through and on the degree of openness. Devereux, Lane and Xu (2006), point the importance of financial constraints on the entrepreneurs who detain the capital stock. The entrepreneurs need an external debt in order to finance their projects, so, the financial accelerator has to be introduced. This type of constraints introduces a higher degree of complexity in our model. In order to get a simpler modelization, our model introduces, as do Laxton and Pesenti (2003) and Adolfson et al (2005), a risk premium on the foreign bond holdings. The imperfect pass-through aspect depends on the value of the parameters of rigidity and the propagation of the shocks in the Phillips curve.

Our goal in this paper, is not only to estimate a DSGE model for transition countries, but also to study the convergence issues of the Central Europe transition economies and the reasons of slower convergence for some countries. We use a small open economy model for five of the Central Europe transition countries (Czech Republic, Hungary, Poland, Slovakia and Slovenia). The choice of countries is because of the fact that they have the largest data samples, the earlier data are in 1995, while they begin in 1999 or 2000 for the remaining countries, but also because these countries have many common characteristics.

To estimate the model we use Bayesian methods, where priors are based other on studies such as Smets and Wouters (2002a and b), Adolfson et al (2005) and Laxton and Pesenti (2003), Then we study the response of the convergence criteria variables to the shocks of the model. This allows us to analyze the reasons of the different dynamics of convergence across countries. We find that Poland still needs to stabilize its economy before its entry to the ERM II, Slovenia undergoes the least shocks effects, Slovakia illusrates less volatility of the convergence criteria because of lower volatilty of shocks, while the remaining countries need to offset the effects of the transition shocks before they join the ERM II.

The rest of the paper is organized as follows: Section 2 presents the model; Section 3 presents the steady state and the calibration of parameters, Section 4 presents the data and the measurement equations, Section 5 presents and interprets the empirical results, Section 6 presents the Ramsey optimal monetary policy and the convergence criteria dynamics. Finally, Section 7 presents the conclusion and further research outlines.

2 The model

2.1 The general structure of the model

We construct a two-sector dynamic stochastic general equilibrium model of a small open economy. Two types of goods are produced locally: a non-tradable good and an export good. Since we consider small open economies, exporters are constrained by the rest of the world demand and prices. As a result, the exports firm is supposed evolving in a perfect competition market. However, the non-tradable goods producers evolve in a monopolistic competition.

There are three sets of domestic actors in the model: households, firms and the monetary authority. Domestic households consume the non-tradable good and a foreign imported good. Firms are of three categories: non-tradable goods producers, tradable goods producers and importers of foreign goods. Export prices follow the law of one price. Importers buy a homogenous good from the foreign market, they transform it into consumption goods and investment goods and sell them to the domestic households.

The model exhibits the following features: a) Calvo type nominal rigidities with partial indexation of non-tradable goods prices, imported goods and wages, b) imperfect pass-through of exchange rate changes into imported goods prices, c) smoothing functions of demand in the form of investment adjustement cost and habit formation of consumption, d) risk premium on foreign bond holdings.

Nominal rigidities are introduced in order to ensure the inertia of prices and wages, and to motivate a role for monetary policy. Imperfect pass-through of exchange rate changes causes deviations from the law of one price. The risk premium is introduced not only to ensure a well-defined steady state as justify Schmitt-Grohé and Uribe (2003), but also due to the fact that this premium risk is an adjustment cost of the external debt, since in small open transition economies investment is constrained by strong financial constraints. Foreign investment is more important in developping countries than in developped countries. As a consequence, investment constraints are very sensible to the terms of trade changes. This fact gives a reason for introducing an important risk premium on foreign debt. We may have different degrees of risk premium and of exchange rate pass-through to import and consumer prices across countries, since the terms of trade are different across transition countries.

The model developped in this section is supposed to have *a priori* the same features and parameters values for the countries of concern.

2.2 Households

The consumption of non-tradable goods and of imported goods are defined over a continuum of differentiated goods

$$C_t = \left[(\gamma^{cn})^{\frac{1}{\theta^{cn}}} \left(C_t^N \right)^{\frac{\theta^{cn}-1}{\theta^{cn}}} + (1-\gamma^{cn})^{\frac{1}{\theta^{cn}}} \left(C_t^F \right)^{\frac{\theta^{cn}-1}{\theta^{cn}}} \right]^{\frac{\theta^{cn}}{\theta^{cn}-1}} \tag{1}$$

$$C_t^N = \left[\int_0^1 \left(C_t^N(z) \right)^{\frac{\varrho-1}{\varrho}} dz \right]^{\frac{\varrho}{\varrho-1}}$$

$$C_t^F = \left[\int_0^1 \left(C_t^F(z^*) \right)^{\frac{\varrho-1}{\varrho}} dz^* \right]^{\frac{\varrho}{\varrho-1}}$$

where ϱ is the elasticity of substitution between the differntiated goods of type z for domestic goods or z^* for foreign goods. The corresponding price indices are as follows:

$$P_t^c = \left[\gamma^{cn} \left(P_t^N \right)^{1-\theta^{cn}} + (1-\gamma^{cn}) \left(P_t^m \right)^{1-\theta^{cn}} \right]^{\frac{1}{1-\theta^{cn}}} \quad (2)$$

$$P_t^N = \left[\int_0^1 \left(P_t^N(z) \right)^{1-\varrho} dz \right]^{\frac{1}{1-\varrho}}$$

$$P_t^m = \left[\int_0^1 \left(P_t^m(z^*) \right)^{1-\varrho} dz^* \right]^{\frac{1}{1-\varrho}}$$

Investment is defined by the same way as consumption, a CES aggregate of non-tradable investment goods and imported investment goods, such we have:

$$I_t = \left[\left(\gamma^{in} \right)^{\frac{1}{\theta^{in}}} \left(I_t^N \right)^{\frac{\theta^{in}-1}{\theta^{in}}} + \left(1-\gamma^{in} \right)^{\frac{1}{\theta^{in}}} \left(I_t^M \right)^{\frac{\theta^{in}-1}{\theta^{in}}} \right]^{\frac{\theta^{in}}{\theta^{in}-1}} \quad (3)$$

$$I_t^N = \left[\int_0^1 \left(I_t^N(z) \right)^{\frac{\varrho-1}{\varrho}} dz \right]^{\frac{\varrho}{\varrho-1}}$$

$$I_t^M = \left[\int_0^1 \left(I_t^M(z^*) \right)^{\frac{\varrho-1}{\varrho}} dz^* \right]^{\frac{\varrho}{\varrho-1}}$$

The corresponding price indices are:

$$P_t^i = \left[\gamma^{in} \left(P_t^N \right)^{1-\theta^{in}} + \left(1-\gamma^{in} \right) \left(P_t^m \right)^{1-\theta^{in}} \right]^{\frac{1}{1-\theta^{in}}} \quad (4)$$

The aggregate investment can also be defined as the aggregate of investment in non-tradable sector and investment in exports sector. While the investment goods aggregate is denoted by I_t^N, the investment in the non-tradable sector is denoted by $I_{N,t}$ and the investment in the exports sector is denoted by $I_{X,t}$

$$P_t^i I_t = P_t^N I_{N,t} + P_t^X I_{X,t} \quad (5)$$

The intertemporal preference of a consumer i is given by

$$U\left(C_t\left(i\right), H_t^s\left(i\right)\right) = E_t \sum_{s=0}^{\infty} \beta^s \left\{ \begin{array}{c} \zeta_t^c \frac{\left(C_{t+s}(i) - bC_{t+s-1}(i)\right)^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}} \\ -\zeta_t^h l_H \frac{\left(H_{t+s}^s(i)\right)^{1+\eta_H}}{1+\eta_H} \end{array} \right\} \quad (6)$$

where β is the discount factor, b is the consumption habit formation coefficient, H_t^s is labor supply in hours worked in the non-tradable sector, σ_c is the elasticity of substitution of consumption, η_H is the inverse of the elasticity of labor supply, $\frac{M_t}{P_t^c}$ are real money balances with θ_M as the elasticity of substitution of real money balances. ζ_t^c and ζ_t^h are, respectively, preference shock of consumption and disutility shock of labor. They follow $AR(1)$ process with ε_t^c and ε_t^h as innovations.

We suppose that the mobility of labor between sectors is perfect, so we can have an equality of wages between sectors, but there is no mobility of labor between countries.

The households maximize their intertemporal utility subject to the budget constraint:

$$\begin{aligned} & P_t^c C_t\left(i\right) + S_t B_{t+1}^*(i) + B_{t+1}(i) \\ = \; & W_t\left(i\right) H_t(i) + S_t(1+R_{t-1}^*)\Phi\left(D_{t-1}, \varphi_t\right) B_t^*(i) \\ & +(1+R_{t-1})B_t(i) + R_{t-1}^{KN} P_t^i K_t^N(i) + R_{t-1}^{KX} P_t^i K_t^X(i) \end{aligned} \quad (7)$$

where $B_t(i)$ $(B_t^*(i))$ are the holdings of bonds, by the household i, denominated in the domestic (foreign) currency, R_t is the nominal interest rate, R_t^* is the foreign nominal interest rate, $S_{j,t}$ is the nominal exchange rate, $R_{N,t}^K$ $\left(R_{X,t}^K\right)$ is the real rental rate of capital used, respectively, in non-tradable (exports) sector, $K_{N,t}(i)$ $(K_{X,t}(i))$ is the capital stock, detained by the household i, used, respectively, in non-tradable (exports) production during the period t. $\Phi\left(D_t, \varphi_t\right)$ is a risk premium function, on foreign bond holdings, which depends on the real foreign debt of the domestic economy in home currency and φ_t is a time-varying shock to the risk premium, it follows an autoregresssive process. The function $\Phi\left(D_t, \varphi_t\right)$ is assumed to be strictly increasing in D_t and to satisfy $\Phi\left(0,0\right) = 1$, we suppose that i has an exponential shape

$$\Phi\left(D_t, \varphi_t\right) = \varphi_t \exp\left(D_t - \bar{D}\right)$$

where we suppose that the steady state debt $\bar{D}$ is zero.

According to Christiano, Eichenbaum and Evans (2005), the capital adjustment, that we adopt, is supposed to follow:

$$K_{N,t+1} = (1-\delta)K_{N,t} + F\left(I_{N,t}, I_{N,t-1}\right) \quad (8)$$

$$K_{X,t+1} = (1-\delta)K_{X,t} + F\left(I_{X,t}, I_{X,t-1}\right) \quad (9)$$

Here, δ denotes the physical rate of depreciation and F is a function that turns investment into capital to be used in the next period. The function F, which is similar across sectors, takes into account the investment adjustment cost::

$$F\left(I_t, I_{t-1}\right) = \left[1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{10}$$

The adjustement cost function has to satisfy the conditions: $\tilde{S}(1) = \tilde{S}'(1) = 0$, and $\tilde{S}''(1) > 0$. These conditions assure that we have a positive convex upward function, so we can use a quadratic function centered around one:

$$\tilde{S}\left(\frac{I_t}{I_{t-1}}\right) = \frac{\mu^s}{2}\left[\frac{I_t}{I_{t-1}} - 1\right]^2 \tag{11}$$

The derivation of the first order conditions and the aggregation over all households give[1]:

$$\frac{\zeta_t^c}{\left(C_t - bC_{t-1}\right)^{\frac{1}{\sigma_c}}} - E_t \frac{\beta b \zeta_{t+1}^c}{\left(C_{t+1} - bC_t\right)^{\frac{1}{\sigma_c}}} = \lambda_t P_t^c \tag{12}$$

Equation (12) assures that the multiplier of the households program equals the real marginal utility of consumption. Using the last equation with the multiplier's growth factor we can get the Euler equation:

$$\frac{\frac{\zeta_t^c}{\left(C_t - bC_{t-1}\right)^{\frac{1}{\sigma_c}}} - \beta b E_t \frac{\zeta_{t+1}^c}{\left(C_{t+1} - bC_t\right)^{\frac{1}{\sigma_c}}}}{E_t \frac{\zeta_{t+1}^c}{\left(C_{t+1} - bC_t\right)^{\frac{1}{\sigma_c}}} - \beta b E_t \frac{\zeta_{t+2}^c}{\left(C_{t+2} - bC_{t+1}\right)^{\frac{1}{\sigma_c}}}} = \beta E_t \frac{1 + R_t}{1 + \pi_{t+1}^c} \tag{13}$$

In order to ensure that the marginal utility is constant at the steady state, we assume, at the steady state, $\frac{1+\bar{R}}{1+\bar{\pi}} = \frac{1}{\beta}$.

The first order conditions with respect to B_{t+1} and B_{t+1}^* give:

$$E_t \frac{S_{t+1}}{S_t} = \frac{1 + R_t}{\Phi\left(D_{t-1}, \varphi_t\right)\left(1 + R_t^*\right)} \tag{14}$$

Equation (14), that is obtained by the equality of earnings from domestic and foreign bond holdings, is an uncovered interest rate parity condition. The first-order condition with respect to the investment $I_{N,t}$ and $I_{X,t}$, by taking into account the investment-capital transmission function $F\left(I_t, I_{t-1}\right)$ gives:

$$\begin{aligned} & R_{N,t}^K E_t \left(F_1'(I_{N,t+1}, I_{N,t}) + F_2'(I_{N,t+1}, I_{N,t}) E_t \frac{1 + \pi_{t+2}^i}{1 + R_{t+1}}\right) \\ = \; & E_t \frac{1 + R_t}{1 + \pi_{t+1}^i} + (1 - \delta) \end{aligned} \tag{15}$$

$$\begin{aligned} & R_{X,t}^K E_t \left(F_1'(I_{X,t+1}, I_{X,t}) + F_2'(I_{X,t+1}, I_{X,t}) E_t \frac{1 + \pi_{t+2}^i}{1 + R_{t+1}}\right) \\ = \; & E_t \frac{1 + R_t}{1 + \pi_{t+1}^i} + (1 - \delta) \end{aligned} \tag{16}$$

[1] For more details, see the appendix

Equations (15) and (16) say that, at equilibrium, the opportunity cost of investment equals the rentability of investment in each sector. The partial derivatives of F are:

$$F_1'(I_t, I_{t-1}) = -\tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) \quad (17)$$

$$F_2'(I_t, I_{t-1}) = \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2 \quad (18)$$

At the steady state, $\tilde{S}\left(\frac{I_t}{I_{t-1}}\right) = \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right) = 0$, this implies that the partial derivatives: $F_1'(I_t, I_{t-1}) = 1$ and $F_2'(I_t, I_{t-1}) = 0$.

The optimal wage condition is:

$$\frac{W_t(i)}{P_t^c} = \frac{\zeta_t^h \left(H_t(i)\right)^{\eta_H}}{\left(\frac{\zeta_t^c}{(C_t - bC_{t-1})^{\frac{-1}{\sigma_c}}} - E_t \frac{\beta b \zeta_{t+1}^c}{(C_{t+1} - bC_t)^{\frac{-1}{\sigma_c}}}\right)} \quad (19)$$

2.2.1 Wage setting

The households optimize their utility to ask for a new wage $\tilde{W}_t(i)$, obtained by equation (19), with a constant probability $(1 - \zeta^w)$. When households do not adjust their optimal wage, wages are indexed by the consumer price inflation to the last period wages, with κ_w as a degree of indexation (*i.e.* a zero degree of indexation means that agents keep the wages level constant when it's not revised)

$$W_t(i) = \left(1 + \pi_{t-1}^c\right)^{\kappa_w} W_{t-1}$$

We suppose that wages are indexed to the last period consumer price inflation, and not to the steady state inflation as do Erceg et al (2000), since the steady state inflation rate may not be observed by the households. So, the aggregate wage level is

$$W_t = \left[\zeta^w \left(\left(1 + \pi_{t-1}^c\right)^{\kappa_w} W_{t-1}\right)^{1-\lambda^w} + (1 - \zeta^w)\left(\tilde{W}_t\right)^{1-\lambda^w}\right]^{\frac{1}{1-\lambda^w}} \quad (20)$$

The CES demand function for labor $H_t(i)$ by the firms is:

$$H_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\lambda^w} H_t \quad (21)$$

where λ^w is the elasticity of substitution between different households labor effort.

As do Erceg et al (2000) to derive the wage setting equation, households optimize their intertemporal utility function with respect to the wage rate, to get the first-order condition for a household who adjusts her wage rate at period

t:

$$E_t \sum_{s=0}^{\infty} (\beta\xi^w)^s \left(\frac{\prod_{k=0}^{s-1} \left(1+\pi_{t+k}^c\right)^{\kappa_w} \tilde{W}_t(i)}{P_{t+s}^c} U_{C,t+s} + U_{H,t+s} \right) H_{t+s}(i) = 0 \quad (22)$$

Combining equation (22) with equation (20) and the demand equation for labor (21), we can get the log-linearized wage setting equation:

$$\begin{aligned} \hat{w}_t &= \frac{\beta}{1+\beta}\left[\hat{w}_{t+1}+\hat{\pi}_{t+1}^c\right] + \frac{1}{1+\beta}\left(\hat{w}_{t-1}+\kappa_w\hat{\pi}_{t-1}^c\right) \\ &\quad + \frac{1+\beta\kappa_w}{1+\beta}\hat{\pi}_t^c - \frac{1}{1+\beta}\frac{(1-\beta\xi^w)(1-\xi^w)}{(1+\lambda^w)\xi^w}\left[\widehat{w}_t - \widehat{MRS}_t\right] \end{aligned} \quad (23)$$

where a variable $\hat{x}_t = \frac{x_t-\bar{x}}{\bar{x}}$, w_t is the real wage rate and MRS is the marginal rate of substitution between labor and consumption $MRS = \frac{-U_H}{U_C}$.

2.3 Firms

There are three categories of firms operating in this open economy; non-tradable firms, importers and exporters. Non-tradable firms produce a differentiated good using capital and labor. Importers import a homogeneous good, bought in the world market, and trasform it into a differentiated good to be selled to the domestic households. Exporters are in a perfect competition market.

2.3.1 Non-tradable goods producers

Non-tradable goods producers produce differentiated non-tradable goods Y_{it}^N using capital $K_{N,it}$, that they rent from households at the rate $R_{N,t}^K$, and by hiring labor from households with W_t^N. The technology used is a Cobb-Douglas function and sells the output to the households for consumption at the price P_{it}^N. In order to take into account the permanent technology development in transition countries, we introduce a unit-root productivity shock z_t

$$Y_{it}^N = A^N \epsilon_t^N \left(z_t H_{it}^N\right)^{1-\alpha^N} \left(K_{N,it}\right)^{\alpha^N} \quad (24)$$

A^N is a scale parameter of the production in the non-tradable sector. ϵ_t^N is a covariance stationary technology shock on the non-tradable market, which equals one at the deterministic state.

$$\epsilon_t^N = \left(1-\rho^{\epsilon^N}\right) + \rho^{\epsilon^N}\epsilon_{t-1}^N + \varepsilon_t^N \quad (25)$$

The non-stationary shock z_t is supposed to follow a random walk without a deterministic trend[2], but we suppose that the growth rate of the permanent

[2] We don't introduce a deterministic trend since we use detrended data in the estimation process. The first estimation results without introducing a permanent shock gave a very persistent technology shock, which indicates the presence of a non-stationary stochastic shock.

shock, denoted by μ_z is an autoregressive process

$$\mu_{z,t} = \rho_{\mu_z} \mu_{z,t-1} + \varepsilon_t^{\mu_z}$$

In order to use a stationarized model we will use, hereafter, the stationarized variables denoted by $\tilde{x}_t = \frac{x_t}{z_t}$. As a result, we get the following stationarized production function

$$\tilde{Y}_{it}^N = A^N \epsilon_t^N \exp\left(-\alpha^N \mu_{z,t}\right) \left(H_{it}^N\right)^{1-\alpha^N} \left(\tilde{K}_{N,it}\right)^{\alpha^N} \tag{26}$$

Y_t^N is a CES aggregator over a continuum of firms in the non-tradable sector

$$Y_t^N = \left[\int_0^1 \left(Y_{it}^N\right)^{\frac{\varrho-1}{\varrho}} di\right]^{\frac{\varrho}{\varrho-1}}$$

where ϱ is the elasticity of substitution between the differentiated goods. So, the price index of the non-tradable goods is

$$P_t^N = \left[\int_0^1 \left(P_{it}^N\right)^{1-\varrho} di\right]^{\frac{1}{1-\varrho}}$$

Then, the CES demand function of the differentiated good is

$$Y_{it}^N = \left(\frac{P_{it}^N}{P_t^N}\right)^{-\varrho} Y_t^N \tag{27}$$

Prices of differentiated goods are supposed to be rigid, such that the probability that a firm resets the price of the differentiated good is constant at each period denoted by $\left(1-\zeta^N\right)$. The probability of non-adjustment ζ^N is a Calvo type. When a firm resets it's price, we denote this price by $\breve{P}_{it}^N$. When prices are not reset, they are indexed by the last period inflation. So, we can write the price index as:

$$P_t^N = \left[\zeta^N \left(\left(1+\pi_{t-1}^N\right)^{\kappa_N} P_{t-1}^N\right)^{1-\varrho} + \left(1-\zeta^N\right)\left(\breve{P}_t^N\right)^{1-\varrho}\right]^{\frac{1}{1-\varrho}} \tag{28}$$

The cost minimization problem facing the firms, subject to the production function, is given by

$$\min_{K_{N,it}, H_{it}^N} W_t H_{it}^N + P_t^i R_{N,t}^K K_{N,it} + \lambda_t^N \left[Y_{it}^N - A^N \epsilon_t^N \left(z_t H_{it}^N\right)^{1-\alpha^N} \left(K_{N,it}\right)^{\alpha^N}\right] \tag{29}$$

where λ_t^N is a Lagrange multiplier but it represents also the nominal marginal cost. The first order conditions of the above problem are:

$$\tilde{W}_t = \lambda_t^N \left(1-\alpha^N\right) A^N \epsilon_t^N \exp\left(-\alpha^N \mu_{z,t}\right) \left(\frac{\tilde{K}_{N,it}}{H_{it}^N}\right)^{\alpha^N} \tag{30}$$

$$P_t^i R_{N,t}^K = \lambda_t^N \alpha^N A^N \epsilon_t^N \exp\left(\left(1-\alpha^N\right)\mu_{z,t}\right) \left(\frac{\tilde{K}_{N,it}}{H_{it}^N}\right)^{\alpha^N - 1} \tag{31}$$

Using the last two equations we can derive the relationship between the rental rate of capital stock, wages and the captial to labor ratio

$$\tilde{W}_t = \frac{1-\alpha^N}{\alpha^N} \exp\left(-\mu_{z,t}\right) \left(\frac{\tilde{K}_{N,it}}{H_{it}^N}\right) P_t^i R_t^{KN} \tag{32}$$

Now, by introducing equation (32) in equation (30) or (31), we can deduce the forme of λ_t^N the nominal marginal cost for domestic non-tradable goods producers:

$$MC_{it}^N = \frac{1}{\epsilon_t^N} \left(P_t^i R_t^{KN}\right)^{\alpha^N} \left(\tilde{W}_t\right)^{1-\alpha^N} \left(\frac{1}{1-\alpha^N}\right)^{1-\alpha^N} \left(\frac{1}{\alpha^N}\right)^{\alpha^N} \tag{33}$$

This equation can be re-written in the real terms:

$$mc_{it}^N = \frac{1}{\epsilon_t^N} \left(\delta_{in,t} R_t^{KN}\right)^{\alpha^N} \left(\tilde{w}_t^N\right)^{1-\alpha N} \left(\frac{1}{1-\alpha^N}\right)^{1-\alpha^N} \left(\frac{1}{\alpha^N}\right)^{\alpha^N} \tag{34}$$

where mc_t is the real marginal cost, δ_t^{in} is the relative price between investment and non-tradable sector and $\tilde{w}_t^N$ is the real stationarized wage level relative to the non-tradable producers price index.

In order to derive the Phillips curve, we will solve the profit maximization problem of the firms. If a firm resets it's price at period t,then it's intertemporal expected profit maximization problem is:

$$\max_{\breve{P}_{it}^N} \left\{ E_t \sum_{s=0}^{\infty} \left(\beta\zeta^N\right)^s \left[\frac{\prod_{k=0}^{s-1} \left(1+\pi_{t+k}^N\right)^{\kappa_N} \left(\breve{P}_{it}^N\right)}{P_{t+s}^N} - mc_{i,t+s}^N \right] Y_{i,t+s}^N \right\} \tag{35}$$

Using the demand function of Y_{it}^N, equation (27), we can write the first order condition of the above problem:

$$\begin{aligned} &\sum_{s=0}^{\infty} \left(\beta\zeta^N\right)^s \left(\prod_{k=0}^{s-1} \left(1+\pi_{t+k}^N\right)^{\kappa_N - 1}\right) \left(\frac{\breve{P}_{it}^N}{P_t^N}\right)^{-\varrho} \\ = &\sum_{s=0}^{\infty} \left(\beta\zeta^N\right)^s \left(\prod_{k=0}^{s-1} \left(1+\pi_{t+k}^N\right)^{\kappa_N - 1}\right) \left(\frac{\breve{P}_{it}^N}{P_t^N}\right)^{1-\varrho} \frac{\varrho}{\varrho - 1} mc_{i,t+s}^N \end{aligned} \tag{36}$$

We replaced the price ratio $\left(\frac{\breve{P}_{it}^N}{P_t^N}\right)$ by that obtained from price index, equation (28), the log-linear form of the first order condition gives a new-keynesian forward and backward looking Phillips curve

$$\hat{\pi}_t^N = \frac{\kappa_N}{1+\beta\kappa_N} \hat{\pi}_{t-1}^N + \frac{\beta}{1+\beta\kappa_N} E_t \hat{\pi}_{t+1}^N + \frac{\left(1-\beta\xi^N\right)\left(1-\zeta^N\right)}{\left(1+\beta\xi^N\right)\zeta^N} \widehat{mc}_t^N \tag{37}$$

where $\hat{\pi}_t = \frac{\pi_t - \bar{\pi}}{\bar{\pi}}$ and $\widehat{mc}_t = \frac{mc_t - \overline{mc}}{\overline{mc}}$

At the steady state the marginal cost is equal to the inverse of the gross mark-up rate, $\overline{mc} = \frac{\varrho - 1}{\varrho}$.

2.3.2 Importers

The imports sector contains firms who import a homogenous product from the foreign market at a price P_t^*. They transform the imported good into differentiated consumption goods (C_{it}^m) or investment goods (I_{it}^m). As we did for non-tradable goods, the probability that an imports firm adjusts it's price is $(1 - \zeta^m)$, when firms do not reset their prices, prices are indexed by the last period inflation. If an imports firm resets it's price at period t,then it's intertemporal expected profit maximization problem for consumption goods importers and investment goods importers, respectevely, are:

$$\max_{\tilde{P}_{it}^{m,c}} \left\{ E_t \sum_{s=0}^{\infty} (\beta \zeta^m)^s \left[\frac{\prod_{k=0}^{s-1} \left(1 + \pi_{t+k}^m\right)^{\kappa_m} \left(\tilde{P}_{it}^m\right)}{P_{t+s}^m} - mc_{t+s}^m \right] \left(C_{i,t+s}^m + I_{i,t+s}^m\right) \right\} \tag{38}$$

The real marginal cost is the real exhange rate RS_t

$$mc_t^m = \frac{S_t P_t^*}{P_t^m} = RS_t \tag{39}$$

Similarly to the non-tradable goods sector, we can derive the Phillips curve

$$\hat{\pi}_t^m = \frac{\kappa_m}{1 + \beta \kappa_m} \hat{\pi}_{t-1}^m + \frac{\beta}{1 + \beta \kappa_m} E_t \hat{\pi}_{t+1}^m + \frac{(1 - \beta \xi^m)(1 - \zeta^m)}{(1 + \beta \xi^m) \zeta^m} \widehat{RS}_t \tag{40}$$

Now, since we have the non-tradable goods inflation equation and the imports inflation rate, we can deduce the consumption price inflation and the investment price inflation, from equations (2) and (4), respectively:

$$\hat{\pi}_t^c = \gamma^{cn} \left(\hat{\delta}_{t-1}^{nc} + \hat{\pi}_t^N \right) + (1 - \gamma^{cn}) \left(\hat{\delta}_{t-1}^{mc} + \hat{\pi}_t^m \right) \tag{41}$$

$$\hat{\pi}_t^i = \gamma^{in} \left(\hat{\delta}_{t-1}^{ni} + \hat{\pi}_t^N \right) + \left(1 - \gamma^{in}\right) \left(\hat{\delta}_{t-1}^{mi} + \hat{\pi}_t^m \right) \tag{42}$$

where δ_t^{ni} (δ_t^{mi}) are, respectively, the relative price of non-tradable goods (imported goods) to investment goods price, and where δ_t^{nc} (δ_t^{mc}) are, respectively, the relative price of non-tradable goods (imported goods) to consumption goods price.

2.3.3 Exporters

Exporters produce a homogenous good in a perfectly competitive market Y_t^X using capital K_t^N, that they rent from households at the rate R_t^{KX}, and by hiring labor from households with W_t^X. The technology used is a Cobb-Douglas function and sells the output in the foreign market, at the price P_t^X, evolving to the one price law:

$$Y_t^X = \epsilon_t^X \left(K_{X,t}\right)^{\alpha^X} \left(H_t^X\right)^{1-\alpha^X} \tag{43}$$

Also a fixed cost is incleded to ensure that profits equal zero at the steady state. ϵ_t^X is a technology shock on the exports market, which equals one at the deterministic state.

$$\epsilon_t^X = \left(1 - \rho^{\epsilon^X}\right) + \rho^{\epsilon^X} \epsilon_{t-1}^X + \varepsilon_{it}^X \tag{44}$$

Since the law of one price holds for exports

$$P_t^X = S_t P_t^* \tag{45}$$

then, the inflation of the exports sector can be written in function of exchange rate evolution and foreign inflation rate

$$\left(1 + \pi_t^X\right) = \frac{S_t}{S_{t-1}} \left(1 + \pi_t^*\right) \tag{46}$$

Similarly to the non-tradable sector, the exporters minimize cost function to get the fifollowing first order conditions

$$\tilde{W}_t = \lambda_t^X \left(1 - \alpha^X\right) \epsilon_t^X \left(\frac{K_{X,it}}{H_{it}^X}\right)^{\alpha^X} \tag{47}$$

$$P_t^i R_{X,t}^K = \lambda_t^X \alpha^X \epsilon_t^X \left(\frac{K_{X,it}}{H_{it}^X}\right)^{\alpha^X - 1} \tag{48}$$

Since exporters are in a perfect competitive market, the marginal cost is equal to one.

The perfect mobility of labor between sectors assumption implies that the nominal wages are equal, but the real wages are related to the relative price between both sectors $\delta_{N,X}$:

$$w_t^X = \tilde{w}_t^N \delta_{N,X,t} \tag{49}$$

2.4 The Central Bank

As done in most of the monetary policy literature, the Central Bank stabilizes the inflation rate, the output gap and the nominal exchange rate using an interest-rate policy instrument which is the interest rate. The interest rate equals the world interest rate at the steady state. The reaction function of the Central Bank has the following form:

$$\frac{1 + R_t}{1 + R^*} = \left(\frac{1 + R_{t-1}}{1 + R_{t-1}^*}\right)^{\gamma} \left(\left(\frac{1 + \pi_t^c}{1 + \bar{\pi}}\right)^{\omega_{\pi_c}} \left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{\omega_Y} \left(\frac{S_t}{S_{t-1}}\right)^{\omega_S}\right)^{1-\gamma} \epsilon_t^m \tag{50}$$

where γ is the inertia degree of the policy rule and ϵ_t^m is an AR(1) stationary process. This is an ordinary Taylor rule but with a term for nominal exhange rate evolution, since in the Maastricht criteria, countries who could join the European Union had to satisfy some stability criteria of exchange rate, inflation rate and output variations, in order to get in the european exchange rate mechanism (ERM II), so then in the Euro area. The ERM II allows for candidate countries for the Euro area to vary their nominal exchange rate 15% with respect to the initial exchange rate at the adoption date of the ERM II.

2.5 The Foreign Economy

The foreign economy represents the Euro area and three other european countries (Danemark, Sweden and UK) who relatively have important trade with the Central Europe countries than the remaining countries of the world. We even ignore the rest of the EU countries for simplicity reasons of the database construction. Since 70-80% of the external trade of the Central Europe transition countries[3] is with the EU-15 countries, we will consider that the major foreign variables fluctuations are from the EU-15 countries.

For a small open economy, the foreign variables are exogenous for the domestic economy. To take into account the fluctuations of the foreign variables we modelize the foreign economy by a simple VAR(2) model including the EU-15 detrended GDP ($\hat{y}_t^*$), consumption inflation ($\hat{\pi}_t^*$)and nominal interest rate $\left(\hat{R}_t^*\right)$.

2.6 Market clearing

The model is closed by imposing the following market clearing condition:

$$P_t^N \tilde{Y}_t^N + P_t^X Y_t^X = P_t^c C_t + P_t^i I_t \tag{51}$$

In real terms, this condition can be written usingrelative prices

$$\tilde{Y}_t^N + \frac{Y_t^X}{\delta_t^{nx}} = \frac{C_t}{\delta_t^{nc}} + \frac{I_t}{\delta_t^{ni}} \tag{52}$$

We denote the external commerce deficit in terms of foreign prices by

$$A_t = \left(C_t^m + I_t^m\right)/\delta_t^{xm} - Y_t^X \tag{53}$$

The external commerce deficit is used to explain the external debt of the home country. The debt is the accumulation of the external commerce deficit

$$D_t = E_{t-1}\frac{(1+R_{t-1}^*)}{(1+\pi_t^*)}D_{t-1} + A_t \tag{54}$$

Since the tradable sector is in perfect competition, the gross markup is closed to one. Thus, relative prices between non-tradable and tradable goods $\delta_t^{nx} = \frac{P_t^N}{P_t^X}$

[3] Czech Republic, Hungary, Slovakia, Slovenia and Poland.

is the gross markup of the non-tradable sector $\frac{\varrho}{\varrho-1}$ at the steady state. The relative price between exports and imports (the real exchange rate) is also the gross markup of the imports sector $\frac{\varrho}{\varrho-1}$ at the steady state. At the short term, the dynamics of relative prices are determined by the ratio between marginal cost of non-tradable and tradable sectors. The relative prices of aggregate prices of consumption or investment to imported or non-tradable goods are obtained by growth forms in term of inflation ratios. All the other relative prices are compounded of δ_t^{nx}, δ_t^{xm}, δ_t^{nc} and δ_t^{ni}.

3 Calibration and steady state

For the elasticity parameters we used values indicated in the litterature, such as Laxton and Pesenti (2003), Adolfson et al (2005) and Chrsitiano, Eichenbaum and Evans (2005). We calibrate the elasticity of intertemporal substitution of consumption σ_c to 10, as used in Laxton and Pesenti (2003) in their calibrated model for the Czech Republic. Also, the inverse elasticity of intertemporal substitution of labor η_H is 0.3, while we find the coefficient l_H 0.48 in order to find the good steady state. The share of total consumption in the output is 60%. The share of home goods in the total consumption is 40%, while the share of home investment goods in total investment is 30%. The elasticity of substitution between non-tradable and imported consumption goods θ^{cn} is 6. We give a lightly lower value of elasticity of substitution between home produced and imported investment goods θ^{cn} equals 5 in both sectors: non-tradable and tradable. The elasticity of substitution between different labor efforts λ_w is calibrated to equal 6. Similarly, the elasticity of substitution between different produced goods by the non-tradable sector or by importers ϱ is 6. This low level of elasticity is supposed in order to impose higher markup and less competitivity in transition countries than developped countries. The share of wages in the value-added of the non-tradable (tradable) sector is, respectively, 71% (65%). The depreciation rate of capital is 2.5%. Finally, the discount factor β is 0.99 which is the inverse of the gross real interest rate, with a steady state inflation rate of 0.9%.

4 Data and measurement issues

To estimate the model we use Eurostat quarterly data for the periods 1996:2-2007:2 for each of the Czech Republic, Hungary, Poland Slovakia and Slovenia. In addition to the foreign variables data, the data variables for each country are GDP, households consumption, investment, consumer price index inflation, nominal interest rate, nominal exchange rate and the nominal wages growth rate. The data variables are detrended by the Hodrick Prescott detrending filter and centered around their means.

Since the observed detrended GDP is non-stationary, we add to the model the following measurement equation that links the stationarized variables to the observed variable containing a unit root

$$\frac{Y_t^{obs}}{Y_{t-1}^{obs}} = \exp\left(\mu_{z,t}\right)\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}}$$

5 The estimation of the model

In order to estimate the DSGE model, we use Bayesian methods, that are based on prior distributions, these priors are updated by the data distribution (the likelihood). In order to compute the likelihood, DYNARE[4] uses the Sims algorithm in order to write the model into a state-space model and then uses the Kalman filter to compute the likelihood. Once we have the likelihood distribution, we can calculate the posterior distribution. Thus, we make any inference about parameters using Metropolis-Hasting iterations from the posterior distributions.

5.0.1 Priors

The priors represent our prior belief about the parameters distribution, which is constructed upon theoretical possible values and previous studies.

The absence of estimated models for the transition countries makes the priors choice a delicate question. Moreover, the small sample size problem motivate us to look for precised priors. In this paper, we used the Smets and Wouters (2002a) priors for the shocks persistence parameters and the indexation degrees. We used uniform distributions on the interval [0,1] as priors of the shocks standard deviation. For monetary policy parameters, we used the Laxton and Pesenti (2003) calibrated values. However, they do not include an exchange rate stabilization objective in their Taylor rule. They state that this coefficient is usually closed to zero, but Lubik and Schorfheide (2007) estimate this parameter for a panel of small open economies, they found it to be around 0.07 for the UK. That's why we use a prior with mean 0.15 which is still a small value but larger than that of the UK since transition countries should satisfy a stabilization criteria of their exchange rates with respect of the Euro. The difficulty is in the choice of the Calvo parameters priors. If we impose the same values used by Smets and Wouters(2002a) or Adolfson et al (2005), this maybe a misspecification, since the high inflation levels in these countries and the transition process may had an important impact to motivate producers to revise more frequently their prices. A second difficulty is that the likelihood initial values are closed to zero using the Smets and Wouters (2002a) priors for the Calvo parameters (prior mean is 0.7). We used the prior of Adolfson et al (2005) for the wages calvo parameter, but we tried different values of the imported and domestic goods Calvo parameters between 0.4 and 0.7. In general lower values for the Calvo parameter of the non-tradable home goods sector $\left(\zeta^N\right)$ give remarkably higher likelihood values. This means that the data show less rigidity

[4] http://www.cepremap.cnrs.fr/dynare

of home-produced goods than imported goods. We retained a prior mean 0.4 for ζ^N and 0.7 for ζ^N. Finally, we used the prior of Adolfson et al (2005) for the consumption habit formation.

5.1 Estimation results

The results show a high degree of persistence of some shocks, notably the stationary productivity shocks in both sectors and the monetary shock (except Poland). This maybe due to the transition process, which had a persistent impact on the productivity of these countries. A persistent monetary shock is expected since the transition process requires from the countries to achieve low inflation closed to the Euro area level and also a stabilization of the exchange rate, even if the monetary shock doesn't exhibit a low volatility.

The unit-root productivity shock is volatile for all the countries, the posterior's mean of the shock's standard daviation is around 0.3, this high volatility explains the high volatility of the data, especially the inflation and the GDP in the second part of the 1990s. The Czech Republic undergoes a highly volatile risk premium shock due to the high openess of its financial market to foreign investment. Preference shocks are also volatile, notably that of Hungary, Poland and Slovenia, they are less volatile for the other two countries which have rather more closed values to those of the Euro area estimated by Adolfson et al (2005). In general, we can observe that Poland and Hungary undergoe the most volatile shocks, while Slovakia has the least volatile shocks among the five transition countries with smaller values of the monetary policy parameters. This fact reveals a question about the entrance of Slovakia in the european exchange rate mechanism (ERM II), it is the only country that could enter this regime after Slovenia among the five countries. The question is if Slovakia could join the ERM II because of its stabilization policy or because of low volatility of shocks during the transition period. We will deal with this question with more details in the next section.

Among the estimated structural parameters, what is common across transition countries is the fact that the estimated consumption's habit formation is closed to one. In addition, we have a high precision degree about this parameter. This result is so different from the value obtained by Adolfson et al (2005) and Smets and Wouters (2002b) for the Euro area, even if we use the same prior as their. Laxton and Pesenti (2003) calibrate this parameter 0.95 in order to explain the hump-shape in consumption data. This high habit of consumption for households in the transition countries means that they care not only about the level of consumption but also about the growth rate.

The adjustment cost of investment, μ_s, is different from a country to another. It is quite high for Slovenia and Hungary, closed to the value of the Euro area estimated by Adolfson et al. The other countries exhibit lower values for this parameter. This implies different dynamics of investment across countries. The markets openess of the transition countries implied a capital inflow into these countries. That's why we observe, in general, less rigidity of investment in transition countries, compared to that of the Euro area.

The data doesn't give much information about the indexation degrees of wages and domestic goods prices, since the posteriors are closed to the priors, but the imported goods prices exhibit a lower degree of indexation in all countries. The imported goods sector exhibits a high level of Calvo nominal rigidity around 0.9 (except Poland). However, domestic goods exhibit relatively much less rigidity of prices than imported goods. This maybe because of the high level of inflation in the beginning of the transition. We can also observe that the nominal rigidity of domestic goods prices are quite different across transition countries. This difference can explain the different volatilities of relative prices and the different degrees of the Balassa-Samuelson effect. The nominal rigidity of wages is quite different across countries, the lowest value of posteriors mean is 0.29 for Slovenia and about 0.65 for the Czech Republic.

The estimated monetary policies illustrate different goals of the central banks. First, the intertia degree γ seems to be different across countries, but in general the HDR include low values of inertia degrees relatively to the estimated parameter for the Euro zone. This fact is because the central banks should react frequently in order to achieve the stabilization objectives. The posterior distributions of the inflation in the Taylor rule ω_{π_c} give higher values than those estimated for the Euro area, but also than the theoretical values usually used in the litterature [1.4,1.7] and even higher than the prior mode, which is the calibrated value of Laxton and Pesenti (2003), except Slovakia and Hungary who have more closed values to Laxton and Pesenti (2003). This means that the inflation stabilization is the main objective of the transition countries in order to achieve their goals of stabilization. The estimated reaction coefficient to the output gap ω_y is however low, relatively to theoretical values used by Laxton and Pesenti (2003), except Poland that has a closer value of the posterior's mean to the prior's mean, which may be explained by the high volatility of GDP in Poland. The estimated reaction functions of the central banks illustrate only a slight significance (except Slovenia) of the nominal exchange rate fluctuations. This joins the debate about the efficiency of reaction to exchange rate movements. The case of Slovenia is quiet different from the remaining countries, since it adopted the Euro in January 2007, and had to fix the exchange rate some time before. That's why we observe a more important coefficient associated to the exchange rate fluctuations ω_S in the Taylor rule.

It is not surprising that we get parameters distributions of Slovenia more closed to that of the Euro zone, since it is known that Slovenia exhibits many closed characteristics with the Euro area countries, which permitted it to join faster than the other entry countries to join the Euro area.

In terms of precision of the estimation results, or equivalently the variance of the posteriors, we can observe that some parameters, notably the indexation parameters and the investment adjustment cost parameter, are dependent on the prior distribution. So, the data does not a tell us alot about these parameters. However, the other posteriors are more informative about parameters than the priors. We observe also a problem of multi-mode posterior distributions for

Hungary's parameters, this can be problematic in the Bayesian inference about these parameters and so a loss of precision on the decision-making. The posteriors of Poland are quite more dependant on priors than the other countries, which means that the data is less informative for Poland than other countries. This result of Poland can be because of the high volatility of the data, it can be a consequence of change in structural parameters or in shocks variance. The shape of the posteriors for Hungary illustrate the presence of several modes, which can also be due to a change in parameters through this period of 10 years. These changes are expected in an atmosphere of transition, openess of markets and a strict stabilization policy of high inflation.

The reduction of the volatility in the transition countries, notably after the year 2000, is not only a result of the economic stability in the whole world but also a result of a structural economic changes in each country. It may be that the stabilization policies of the central banks could achieve the objectives of volatility reduction, but it may be also that these countries were less exposed to shocks when they joined the EU25. It will be interesting to distinguish between the reasons of convergence: a stabilization policy or less volatilty of shocks. Even if we try to study this question analytically in the next section, it is important to study it in a more advanced method trying to analyze for each parameter and shock the presence of a change point. We will study this feature in the further research in a regime switching DSGE framework.

6 The convergence citeria under the Ramsey optimal policy

In order to compare the convergence criteria between countries, we analyze the impulse response functions of the shocks for the Ramsey allocation of a second order approximation of the model with a timeless perspective *à la* Benigno and Woodford (2006) (*i.e*: the allocation that maximizes the households welfare under the equilibrium conditions assuming that at the initial period the variables satisfy the equilibrium conditions). The program is

$$\max E_t \sum_{s=0}^{\infty} \beta^s \left\{ \begin{array}{c} \zeta_t^c \frac{\left(C_{t+s}(i) - bC_{t+s-1}(i)\right)^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}} \\ -\zeta_t^h l_H \frac{\left(H_{t+s}^s(i)\right)^{1+\eta_H}}{1+\eta_H} \end{array} \right\}$$

subject to the equilibrium equations (13), (14), (15), $(\mathbf{??})$, (23), (26), (34), (37), (40), (41), (42), (43), (46), (49), (50) and the clearing conditions $(52) - (54)$.

First, we study the impulse response functions of the convergence criteria[5] to the different shocks of the model, using the mean values of the estimated shocks standard deviation posteriors. We notice different convergence degrees across

[5]The ERM II criteria require from the candidate countries to stabilize their inflation, output gap, the external debt and the nominal exchange rate.

the transition countries. Poland is the country that has the most volatile criteria, even if the estimated monetary rule has posteriors with high stabilization coefficients. Actually, Poland is the country that had to undergo many volatile shocks during the transition. Even if Poland began to stabilize its economy recently, it is still far from the access to the european monetary union criteria. The Czech Republic seems to perform better than Poland, except in the case of a risk premium shock, which is estimated to be volatile for the Czech Republic. Thus the Czech Republic can be one of the possible future candidates to the ERM II if it undergoes less volatile risk premium shocks. The country that stabilizes its economy the most is Slovenia, the responses to shocks are more convergent than the other countries. That's why Slovenia could join the Euro area in January 2007. For Hungary and Slovakia, they are between the cases of Poland and Slovenia. Hungary's responses are a little bit oscillatory, this is due to the high degree of the investment adjustment cost, which introduces complex roots in the model. Slovakia accessed to ERM II, but did not adopt yet the Euro, the stabilization criteria seem to be satisfied by Slovakia regarding the IRFs. As a result, it is optimal for Slovenia and Slovakia to stabilize their exchange rate, while it is rather optimal for Poland to keep its exchange rate flexible. For the Czech Republic and Hungary the optimal policy is to stabilize inflation more than nominal exchange rate. This is obvious in the IRFs where the response of inflation is less volatile than the response of exchange rate growth.

This result is consistent with Devereux (2003), where he points that it is preferable for transition countries to react to nominal exchange rate movements if the country exhibits sticky prices and wages, since prices and wages do not respond immediately to exchange rate movements, while the demand adjusts immediately after a capital inflow in the case of a risk premium shock. However, in our model, we have another dimension of adjustment, which is the habit formation of consumption. The higher degree of consumption habit formation the less demand responds immediately after the shock, which is obvious in the reaction of the external debt of Poland (the highest degree of consumption habit). That's why it is less optimal for Poland and Hungary to fix their nominal exchange rates than the other countries who exhibit less degree of consumption habit.

Here, we reveal the question that we asked in the last section, which is if Slovakia could join the ERM II because it could stabilize its economy or because it undergoes less volatile shocks. To analyze that, we study the impulse response functions of the convergence criteria to 1% shock in each country. The result is that Poland is still the country that has the most volatile convergence criteria , notably due to a unit-root productivity shock, but Slovakia exhibits more volatility than the other three countries due to a monetary shock, a preference shock or a risk premium shock. This indicates that the transition process did not introduce highly volatile shocks to the slovak economy. Thus, Slovakia could join the ERM II even if the central bank reacts less viloently to inflation, output gap and nominal exchange rate movements. The Czech Republic and Hungary seem to satisfy the convergence criteria better than Slovakia after 1% shocks. So, the Czech Republic and Hungary are possible future candidates to the ERM

II once they offset the transition shocks. Slovenia is still to be the country with the least volatile convergence criteria, with 1% shocks. So, we can conclude that Slovenia satisfies the convergence criteria because it stabilizes its economy.

7 Conclusion

To conclude this paper, we point the most important results: First, households consume with a high habit formation in transition countries, which permits to get a hump-shape response function of consumption and output to a monetary shock, which is more compatible with the data. Second, the imperfect pass-through of the exchange rate movements is highly present in these countries. The estimated monetary rule targets highly the inflation fluctuations, specially the Czech Republic, Poland and Slovenia. The estimated monetary policies indicate that the central banks of transition countries react only slightly to nominal exchange rate fluctuations, except Slovenia who had to fix its nominal exchange rate in order to adopt the Euro.

We analyzed the Ramsey optimal monetary policy and the convergence criteria to join the ERM II. We observe that Slovenia performs well in stabilizing its economy. While Poland exhibits volatile criteria, Slovakia that joined the ERM II performs better than the remaining countries, not due to its stabilization policy but to low volatile shocks that attack the economy. The Czech Republic and Hungary can be future candidates if they offset the highly volatile shocks of the economy. The optimal policy using the estimated parameters and standard deviation of shocks features that it is optimal for Slovenia and quietly for Slovakia to stabilize the nominal exchange rate, while the optimal monetary policy for the Czech Republic and Hungary is to target more the inflation rather than the nominal exchange rate growth.

Even if the model used in this paper explains many of the transition countries features, it will be preferable if we can expand it to include other characteristics. Some papers based on Bernanke, Gertler and Gilchrist (1999) do not suppose that capital is detained by households, but rather by entrepreneurs and firms have access to the credit market and face financial constraints. This aspect may be more descriptive of reality in emerging countries and may better explain the investment dynamics. Another important aspect is to introduce a fiscal policy role, this aspect can be useful in explaining some movements, especially when the monetary policy is constrained to the European Central Bank's policy, the fiscal policy can be an instrument to stabilize the transition economies and notably the debt.

Two other extensions are interesting to develop in the estimation approach: First, to use the result of Sadeq (2008), where we pointed the precision gain of the use of the panel approach if we can find some common characteristics across countries. Since we found that the habit formation is high for all countries and that the values of the Calvo parameter in the imports sector are closed, we can do a pooled estimation, supposing that these two parameters are common across countries and supposing that the countries have also global common shocks. However, this approach is time-consuming, it may be feasible for simpler DSGE models. The second extension, that can be useful in the analysis whether convergence is obtained by policy change or by less volatile shocks, is to estimate a nonlinear DSGE model or a regime-switching model, where parameters are supposed to change over time. This aspect is left for further research.

References

[1] Adolfson, M., Laseén, S., Lindé, J. and M. Villani (2005), "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through", Working paper no.179, Sveriges Riksbank.

[2] Alvarez, J. and M. Arellano (2003), "The Time Series and Cross-Section Asymptotics of Dynamic Panel Data Estimators", Econometrica, vol 71, no. 4, pp 1121-1159.

[3] Anderson G. (2000), "A Reliable and Computationally Efficient Algorithm for Imposing the Saddle Point Property in Dynamic Models", Board of Governors of the Federal Reserve System.

[4] Anderson G. (2006), "Solving Linear Rational Expectations Models: A Horse Race", Board of Governors of the Federal Reserve System.

[5] Antal, J. and F. Brazdik (2006), "Exchange rate peg versus inflation targeting prior to a monetary union entry", working paper, CERGE-EI.

[6] Jazbec B. (2002), "Balassa-Samuelson Effect in Transition Economies: The Case of Slovenia", William Davidson Working Paper Number 507.

[7] Battini, N., Harrison, M., and S.P. Millard (2003), "Monetary policy rules for an open economy", Journal of Economic Dynamics & Control, volume 27, pp 2059 – 2094

[8] Bayoumi, T., D. Laxton and P. Pesenti (2004), "Benefits and spillovers of greater competition in Europe: A macroeconomic assessment", working paper 341, European Central Bank.

[9] Benigno, P. and M. Woodford (2006), "Linear-Quadratic approximation of optimal policy problems" NBER working paper no. 12672

[10] Bernanke B., Gertler M. and S. Gilchrist (1999), "The financial accelerator in a quantitative business cycle framework", Handbook of Macroeconomics, in J.B. Taylor & M. Woodford (ed), edition 1, volume 1, chapter 21, pp 1341 1393.

[11] Calvo G. (1983) "Staggered Prices in a Utility-Maximizing Framework", Journal of Monetary Economics, vol 12, pp 383-398.

[12] Canova, F., Ciccarelli, M. and E. Ortega (2004), "Similarities and Convergence in G-7 Cycles".

[13] Chang, Y., Gomes, J. and F. Schorfheide (2002), "Learning by doing", The American Economic Review, vol. 92, no. 5, pp 1498-1520.

[14] Christiano, L.J., Eichenbaum, M. and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", Journal of Political Economy, vol. 113, no. 1.

[15] Ciccarelli, M. and A. Rebucci (2003), "Has The Transmission Mechanism of European Monetary Policy Changed Over Time?"

[16] Corsetti G., Dedola L. and S. Leduc (2005), "DSGE Models of High Exchange-Rate Volatility and Low Pass-Through", International Finance Discussion Papers Number 845, Board of Governors of the Federal Reserve System.

[17] De Paoli B. (2006), "Monetary Policy and Welfare in a Small Open Economy", CEP Discussion Paper No 639.

[18] Devereux, M. and C. Engels (2003), "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibilty", Review of Economic Studies, 70, pp 765-784.

[19] Devereux, M., Lane, P. and J. Xu (2005), "Exchange Rates and Monetary Policy in Emerging Market Economies".

[20] Erceg, C., Henderson D. and A. Levin (2000), "Optimal monetary policy with staggered wage and price contracts", Journal of Monetary Economics, vol 46, pp 281-313.

[21] Gali J., Gertler, M. and D. Lopez-Salido (2001), "European inflation dynamics". European Economic Review 45, pp 1237–1270.

[22] Geweke, J. (2005), "*Contemporary Bayesian Econometrics and Statistics*", Chapters 3 & 4, WILEY editions, UK.

[23] Harvey, A. (1989), "*Forecasting, structural time series models and the Kalman filter*", Chapter 3, Cambridge University Press.

[24] Hlédik, T. (2003), "A calibrated structural model of the Czech economy", Discussion paper no. 35, Bank of Finland.

[25] Hsiao, C. (2002), "*Analysis of Panel Data*", second edition, Chapters 4 & 9, Cambridge University Press.

[26] Justiniano, A. and M. Kumhof (2006), "Multi-Sectoral Cascading and Price Dynamics - A Bayesian Econometric Evaluation"

[27] Koop, G., Osiewalski, J. and M. Steel (2000), "Modeling the sources of Output growth in a panel of countries", Journal of Business & Economic Statistics.

[28] Koop, G. (2003), "*Bayesian Econometrics*", Chapters 7 & 8, WILEY editions, UK.

[29] Koop, G., Leon-Gonzalez, R. and R. Strachan (2006), "Bayesian Inference in a Cointegrating Panel Data Model", working paper no. 06/2, University of Leicester, UK.

[30] Laxton, D. and P. Pesenti (2003), "Monetary rules for small, open, emerging economies" Journal of Monetary Economics, vol 50, pp 1109–1146

[31] Lubik, L. and F. Schorfheide (2007), "Do central banks respond to exchange rate movements? A structural investigation" Journal of Monetay Economics, vol 54, pp 1069-1087

[32] Murchison, S. (2004), "Exchange Rate Pass-through and Monetary Policy: How Strong is the Link?"

[33] Natalucci, F. and F. Ravenna (2002), "The Road to Adopting the Euro: Monetary Policy and Exchange Rate Regimes in EU Candidate Countries", discussion paper no. 741, Board of Governors of the Federal Reserve System.

[34] Peng, Z. and D. Small, "Bayesian Inference for Random Coefficient Dynamic Panel Data Models", Department of Statistics, The Wharton School, University of Pennsylvania.

[35] Pytlarczyk, E. (2005), "An estimated DSGE model for the German economy within the euro area", Discussion Paper Series 1: Economic Studies No 33/2005, Deutsche Bundesbank.

[36] Sadeq T. (2008), "Bayesian Analysis of DSGE models: a Panel Approach", working paper no. 08-03, EPEE, Université d'Evry Val d'Essonne.

[37] Sahuc, J.G. (2004), "Partial Indexation, Trend Inflation and the Hybrid Phillips Curve", working-paper no. 04-05, EPEE.

[38] Shmitt-Grohé, S. and M. Uribe (2002), "Closing Small Open Economy Models", NBER working paper no. 9270

[39] Smets, F. and R. Wouters (2002a), "Openness, imperfect exchange rate pass-through and monetary policy", Journal of Monetary Economics, vol 49, pp 947–981

[40] Smets, F. and R. Wouters (2002b), "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area", working paper no. 171, European Central Bank.

8 Appendix:

8.1 Model equations

8.1.1 Households first order conditions

Let's write the Lagrange function:

$$\begin{aligned}
\pounds_t &= E_t \sum_{s=0}^{\infty} \beta^s \left\{ \begin{array}{c} \zeta_t^c \left(C_{t+s}(i) - bC_{t+s-1}(i) \right)^{1-\frac{1}{\sigma_c}} \\ -\zeta_t^h l_H \frac{\left(H_{t+s}^s(i)\right)^{1+\eta_H}}{1+\eta_H} \end{array} \right\} \\
&+ E_t \sum_{s=0}^{\infty} \beta^s \lambda_{t+s} \left[\begin{array}{c} P_{t+s}^c C_{t+s}(i) + S_{t+s+1} B_{t+s+1}^*(i) + B_{t++s1}(i) \\ +P_{t+s}^i I_{t+s}(i) - W_{t+s}(i) H_{t+s}(i) \\ -S_{t+s}(1+R_{t+s-1}^*) \Phi \left(D_{t+s-1}, \varphi_{t+s-1} \right) B_{t+s}^*(i) \\ -(1+R_{t+s-1}) B_{t+s}(i) - R_{t+s-1}^{KN} P_{t+s}^i K_{t+s}^N(i,j) \\ -R_{t+s-1}^{KX} P_{t+s}^i K_{t+s}^X(i) \end{array} \right] \\
&+ E_t \sum_{s=0}^{\infty} \beta^s q_{t+s}^N \left[K_{N,t+s+1} - (1-\delta) K_{N,t+s} - F\left(I_{N,t+s}, I_{N,t+s-1} \right) \right] \\
&+ E_t \sum_{s=0}^{\infty} \beta^s q_{t+s}^X \left[K_{X,t+s+1} - (1-\delta) K_{X,t+s} - F\left(I_{X,t+s}, I_{X,t+s-1} \right) \right]
\end{aligned}$$

The first order conditions are:

$$\frac{\partial \pounds_t}{\partial C_t(i)} = \frac{\zeta_t^c}{\left(C_t(i) - bC_{t-1}(i) \right)^{\frac{-1}{\sigma_c}}} - \beta b E_t \frac{\zeta_{t+1}^c}{\left(C_{t+1}(i) - bC_t(i) \right)^{\frac{-1}{\sigma_c}}} - \lambda_t P_t^c = 0 \tag{A.1.1}$$

$$\frac{\partial \pounds_t}{\partial H_t(i)} = -\zeta_t^h l_H \left(H_t(i) \right)^{\eta_H} + \lambda_t W_t(i) = 0 \tag{A.1.2}$$

$$\frac{\partial \pounds_t}{\partial B_{t+1}(i)} = -\lambda_t + \beta \left(1 + R_t \right) E_t \lambda_{t+1} = 0 \tag{A.1.3}$$

$$\frac{\partial \pounds_t}{\partial B_{t+1}^*(i)} = -S_t \lambda_t + \beta \left(1 + R_t^* \right) E_t S_{t+1} \lambda_{t+1} = 0 \tag{A.1.4}$$

In order to derive the first-order condition of investment, taking into account the capital accumulation equation, we will derive the intertemporal utility function subject to the budget constraint, equation (7), and the capital accumulation equation in each sector, equations (8) and (9). The Lagrange multiplier of the budget constraint is λ_t, while the Lagrange multipliers of the capital accumulation are q_t^N and q_t^X.

$$\frac{\partial \pounds_t}{\partial I_{N,t}(i)} = -\lambda_t P_t^i + q_t^N F_{1,t} + \beta E_t q_{t+1}^N F_{2,t+1} = 0 \tag{A.1.6}$$

$$\frac{\partial \pounds_t}{\partial I_{X,t}(i)} = -\lambda_t P_t^i + q_t^X F_{1,t} + \beta E_t q_{t+1}^X F_{2,t+1} = 0 \tag{A.1.7}$$

Deriving with respect to capital $K_{N,t+1}$ and $K_{X,t+1}$ gives:

$$\frac{\partial \pounds_t}{\partial K_{N,t+1}(i)} = \beta E_t \lambda_{t+1} P^i_{t+1} R^K_{N,t} - q^N_t + \beta E_t q^N_{t+1}(1-\delta) = 0$$
$$\frac{\partial \pounds_t}{\partial K_{X,t+1}(i)} = \beta E_t \lambda_{t+1} P^i_{t+1} R^K_{X,t} - q^X_t + \beta E_t q^X_{t+1}(1-\delta) = 0$$

Equation (A.1.3) gives that

$$\frac{\lambda_t}{E_t \lambda_{t+1}} = \beta (1 + R_t) \quad \text{(A.1.8)}$$

The combination of the FOCs wrt capital with each (A.1.6) and (A.1.7) and equation (A.1.8) gives the first-order conditions with respect to investment, equations (15) and (**??**) in the core of the model.

Combining equation (A.1.8) with the same equation obtained from equation (A.1.4) gives equation (14). Equation (A.1.1) gives the value of the multiplier:

$$\lambda_t = \left(\frac{\zeta^c_t}{(C_t(i) - bC_{t-1}(i))^{\frac{-1}{\sigma_c}}} - \frac{\beta b \zeta^c_{t+1}}{(C_{t+1}(i) - bC_t(i))^{\frac{-1}{\sigma_c}}} \right) \frac{1}{P^c_t} \quad \text{(A.1.9)}$$

Using the ratio of the marginal utility $\frac{U_{C_t}}{U_{C_{t+1}}}$ and equation $(A.1.8)$ gives the Euler equation (13) and the optimal wage equation (19).

Finally, introducing conditions (A.1.6) and (A.1.7) in the first-order conditions of capital, with the fact that $\frac{q_t}{q_{t+1}} = \beta \frac{1+R_t}{1+E_t \pi^i_{t+1}}$, will give the first-order conditions of investment, equations (15) and (**??**).

8.1.2 Wage setting equation

If we divide equation (22) by $U_{C,t}$, we will get

$$E_t \sum_{s=0}^{\infty} (\beta \xi^w)^s \left(\frac{\prod_{k=0}^{s-1} \left(1 + \pi^c_{t+k}\right)^{\kappa_w} \tilde{W}_t(i)}{P^c_{t+s}} - MRS_{t+s} \right) H_t(i) = 0 \quad \text{(A.2.1)}$$

where MRS_t is the marginal rate of substitution between labor and consumption $MRS_t = \frac{U_{H,t}}{U_{C,t}}$. Using the demand function of labor, equation (21), we can replace $H_t(i)$ by it's demand, equation $(A.2.1)$ becomes:

$$E_t \sum_{s=0}^{\infty} (\beta \xi^w)^s \left(\begin{array}{c} \left(\frac{\tilde{W}_t(i)}{W_t}\right)^{-\lambda^w + 1} \left(\prod_{k=0}^{s-1} \left(1 + \pi^c_{t+k}\right)^{\kappa_w - 1} \right) \frac{\tilde{W}_t(i)}{P^c_t} \\ - \left(\frac{\tilde{W}_{t+s}(i)}{W_{t+s}}\right)^{-\lambda^w} MRS_{t+s} \end{array} \right) H_{t+s} = 0$$

This is equivalent to

$$E_t \sum_{s=0}^{\infty} (\beta \xi^w)^s \left(\begin{array}{c} \left(\frac{\breve{W}_t(i)}{W_t}\right)^{1-\lambda^w} \left(\prod_{k=0}^{s-1} \left(1 + \pi^c_{t+k}\right)^{\kappa_w - 1} \right) \frac{W_t}{P^c_t} \\ - \left(\frac{\breve{W}_{t+s}(i)}{W_{t+s}}\right)^{-\lambda^w} MRS_{t+s} \end{array} \right) H_{t+s} = 0$$

which is equivalent to

$$
\begin{aligned}
& E_t \sum_{s=0}^{\infty} (\beta\xi^w)^s \left(\prod_{k=0}^{s-1} \left(1+\pi_{t+k}^c\right)^{\kappa_w - 1} \right) \left(\frac{\breve{W}_t(i)}{W_t} \right)^{-\lambda^w + 1} \frac{W_t}{P_t^c} H_{t+s} \qquad \text{(A.2.2)} \\
= \;& E_t \sum_{s=0}^{\infty} (\beta\xi^w)^s \left(\frac{\breve{W}_{t+s}(i)}{W_{t+s}} \right)^{-\lambda^w} MRS_{t+s} H_{t+s}
\end{aligned}
$$

We can get the ratio $\frac{\tilde{W}_t(i)}{W_t}$ from equation (20)

$$
\left(\frac{\breve{W}_t\,(i)}{W_t} \right)^{1-\lambda^w} = \left[\frac{1}{1-\xi^w} - \frac{\xi^w}{1-\xi^w} \left(\frac{\left(1+\pi_{t-1}^c\right)^{\kappa_w} W_{t-1}}{W_t} \right)^{1-\lambda^w} \right] \qquad \text{(A.2.3)}
$$

Hereafter, we will denote the real wage by w_t. So, equation (55) becomes

$$
\begin{aligned}
& \left[\frac{1}{1-\xi^w} - \frac{\xi^w}{1-\xi^w} \left(\frac{\left(1+\pi_{t-1}^c\right)^{\kappa_w}}{\frac{w_t}{w_{t-1}}\left(1+\pi_t^c\right)} \right)^{1-\lambda^w} \right]^{\frac{1}{1-\lambda^w}} \qquad \text{(A.2.4)} \\
& \times w_t E_t \sum_{s=0}^{\infty} (\beta\xi^w)^s \left(\prod_{k=0}^{s-1} \left(1+\pi_{t+k}^c\right)^{\kappa_w - 1} \right) \\
= \;& \frac{1}{1-\beta\xi^w F} \left[\frac{1}{1-\xi^w} - \frac{\xi^w}{1-\xi^w} \left(\frac{\left(1+\pi_{t-1}^c\right)^{\kappa_w}}{\frac{w_t}{w_{t-1}}\left(1+\pi_t^c\right)} \right)^{1-\lambda^w} \right]^{\frac{-\lambda^w}{1-\lambda^w}} MRS_t
\end{aligned}
$$

where F is a Lead operator. So, we get a backward-forward looking equation

$$
\begin{aligned}
& w_t E_t \sum_{s=0}^{\infty} (\beta\xi^w)^s \left(\prod_{k=0}^{s-1} \left(1+\pi_{t+k}^c\right)^{\kappa_w - 1} \right) \\
& -\beta\xi^w E_t w_{t+1} \sum_{s=0}^{\infty} (\beta\xi^w)^s \left(\prod_{k=0}^{s-1} \left(1+\pi_{t+k+1}^c\right)^{\kappa_w - 1} \right) \\
= \;& \left[\frac{1}{1-\xi^w} - \frac{\xi^w}{1-\xi^w} \left(\frac{\left(1+\pi_{t-1}^c\right)^{\kappa_w}}{\frac{w_t}{w_{t-1}}\left(1+\pi_t^c\right)} \right)^{1-\lambda^w} \right]^{\frac{-\lambda^w}{1-\lambda^w}} MRS_t
\end{aligned}
$$

The log-linearized form of the last equation gives the wage setting equation (23)

8.1.3 The Phillips curve derivation

The first order condition, equation (36), of the profit maximization problem (35) is equivalent to

$$\sum_{s=0}^{\infty}\left(\beta\zeta^{N}\right)^{s}\left(\prod_{k=0}^{s-1}\left(1+\pi_{t+k}^{N}\right)^{\kappa_N-1}\right) \tag{A.3.1}$$

$$= \sum_{s=0}^{\infty}\left(\beta\zeta^{N}\right)^{s}\left(\prod_{k=0}^{s-1}\left(1+\pi_{t+k}^{N}\right)^{\kappa_N-1}\right)\left(\frac{\breve{P}_{it}^{N}}{P_t^N}\right)\frac{\varrho}{\varrho-1}mc_{t+s}^{N}$$

The ratio $\left(\frac{\tilde{P}_t^N}{P_t^N}\right)$ can be obtained from equation (28)

$$\left(\frac{\tilde{P}_t^N}{P_t^N}\right)^{1-\varrho} = \left[\frac{1}{1-\xi^N}-\frac{\xi^N}{1-\xi^N}\left(\frac{1+\pi_{t-1}^N}{1+\pi_t^N}\right)^{1-\varrho}\right] \tag{A.3.2}$$

Similarly to the wage setting, equation (55) becomes

$$E_t\sum_{s=0}^{\infty}\left(\beta\zeta^{N}\right)^{s}\left(\prod_{k=0}^{s-1}\left(1+\pi_{t+k}^{N}\right)^{\kappa_N-1}\right) \tag{A.3.3}$$

$$= \left[\frac{1}{1-\xi^N}-\frac{\xi^N}{1-\xi^N}\left(\frac{1+\pi_{t-1}^N}{1+\pi_t^N}\right)^{1-\varrho}\right]^{\frac{1}{1-\varrho}}$$

$$\times\frac{\varrho}{\varrho-1}E_t\sum_{s=0}^{\infty}\left(\beta\zeta^{N}\right)^{s}\left(\prod_{k=0}^{s-1}\left(1+\pi_{t+k}^{N}\right)^{\kappa_N-1}\right)mc_{t+s}^{N}$$

This can be written as follows

$$\frac{\varrho-1}{\varrho}\left[\frac{1}{1-\xi^N}-\frac{\xi^N}{1-\xi^N}\left(\frac{1+\pi_{t-1}^N}{1+\pi_t^N}\right)^{1-\varrho}\right]^{\frac{-1}{1-\varrho}} \tag{A.3.4}$$

$$\times E_t\sum_{s=0}^{\infty}\left(\beta\zeta^{N}\right)^{s}\left(\prod_{k=0}^{s-1}\left(1+\pi_{t+k}^{N}\right)^{\kappa_N-1}\right) = E_t\frac{1}{1-\beta\zeta^N F}mc_t^N$$

where F is the lead operator.

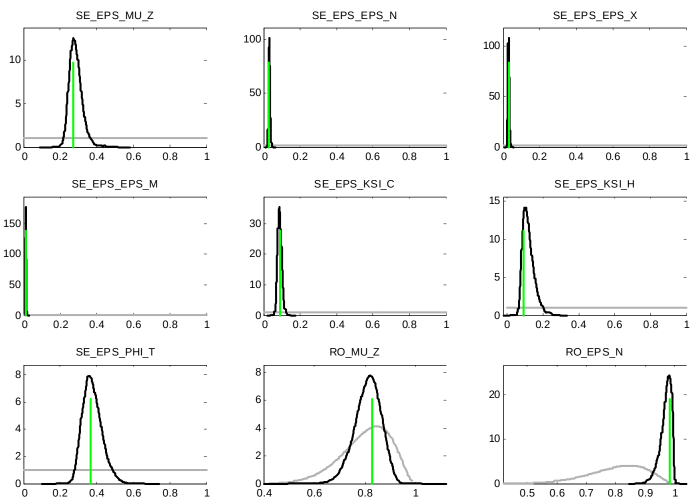
The log-linearization of equation (A.3.4) gives the backward-forward looking Phillips curve equation (37). More details of the derivation of the Phillips curve and the linearization are available in Sahuc (2004).

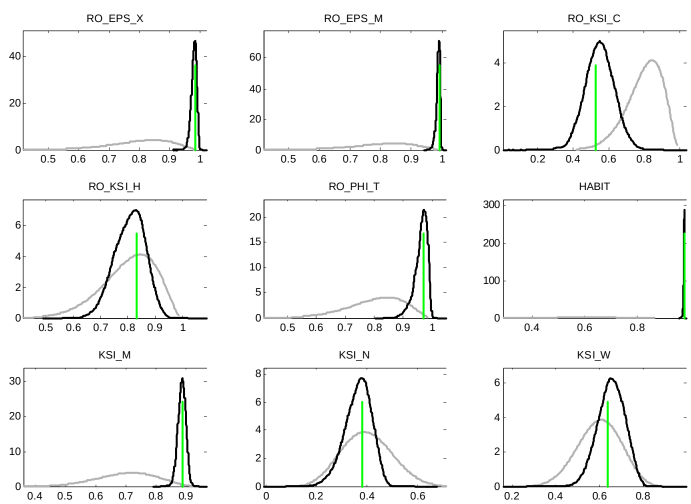
8.2 Estimation results

Table 1: Estimation results of the Czech Republic

Param.	prior	prior mean	prior std	post. mode	post. mean	HDR (90%)	
ρ_{μ_z}	beta	0.8	0.1	0.8290	0.8097	0.7270	0.8974
ρ_{ϵ^N}	beta	0.8	0.1	0.9819	0.9695	0.9416	0.9969
ρ_{ϵ^X}	beta	0.8	0.1	0.9830	0.9802	0.9665	0.9941
ρ_{ϵ^m}	beta	0.8	0.1	0.9922	0.9887	0.9797	0.9979
ρ_{ξ^c}	beta	0.8	0.1	0.5254	0.5483	0.4146	0.6774
ρ_{ξ^H}	beta	0.8	0.1	0.8330	0.8065	0.7155	0.8950
ρ_{φ}	beta	0.8	0.1	0.9723	0.9619	0.9297	0.9939
b	beta	0.6	0.1	0.9800	0.9774	0.9743	0.9800
ξ^m	beta	0.7	0.1	0.8895	0.8861	0.8651	0.9071
ξ^N	beta	0.4	0.1	0.3792	0.3637	0.2808	0.4514
ξ^w	beta	0.6	0.1	0.6413	0.6530	0.5533	0.7565
κ^m	beta	0.4	0.1	0.1432	0.1370	0.0748	0.1968
κ^N	beta	0.4	0.1	0.3872	0.4040	0.2400	0.5641
κ^w	beta	0.4	0.1	0.3827	0.3954	0.2269	0.5555
μ_s	normal	5.0	1.5	4.9402	4.7001	2.2804	7.1195
γ	beta	0.2	0.1	0.1062	0.1362	0.0337	0.2307
ω_{π_c}	normal	1.78	0.3	1.8060	1.8823	1.5372	2.2269
ω_S	normal	0.15	0.05	0.0223	0.0202	-0.0341	0.0723
ω_y	normal	0.6	0.2	0.0843	0.0966	0.0537	0.1390
$\sigma_{\varepsilon\mu_z}$	uniform	0.5	0.2887	0.2714	0.2849	0.2300	0.3388
σ_{ε^N}	uniform	0.5	0.2887	0.0275	0.0301	0.0236	0.0365
σ_{ε^X}	uniform	0.5	0.2887	0.0279	0.0291	0.0226	0.0351
σ_{ε^m}	uniform	0.5	0.2887	0.0087	0.0097	0.0059	0.0133
σ_{ε^c}	uniform	0.5	0.2887	0.0881	0.0868	0.0674	0.1051
σ_{ε^h}	uniform	0.5	0.2887	0.0968	0.1213	0.0731	0.1708
σ_{φ}	uniform	0.5	0.2887	0.3690	0.3757	0.2909	0.4590

Prior and posterior distributions of the Czech Republic parameters





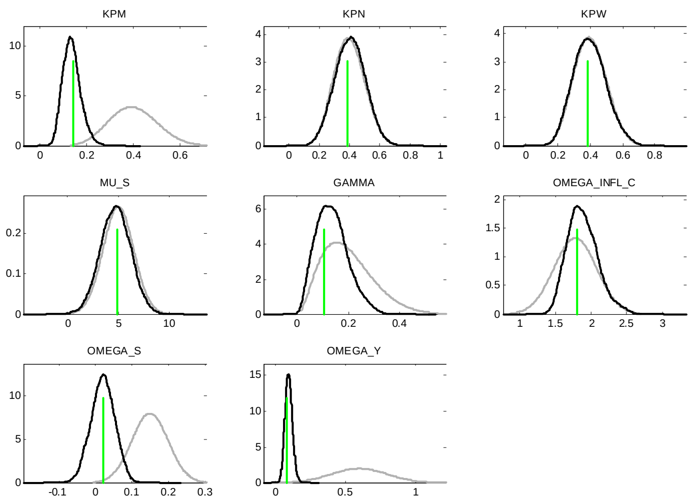
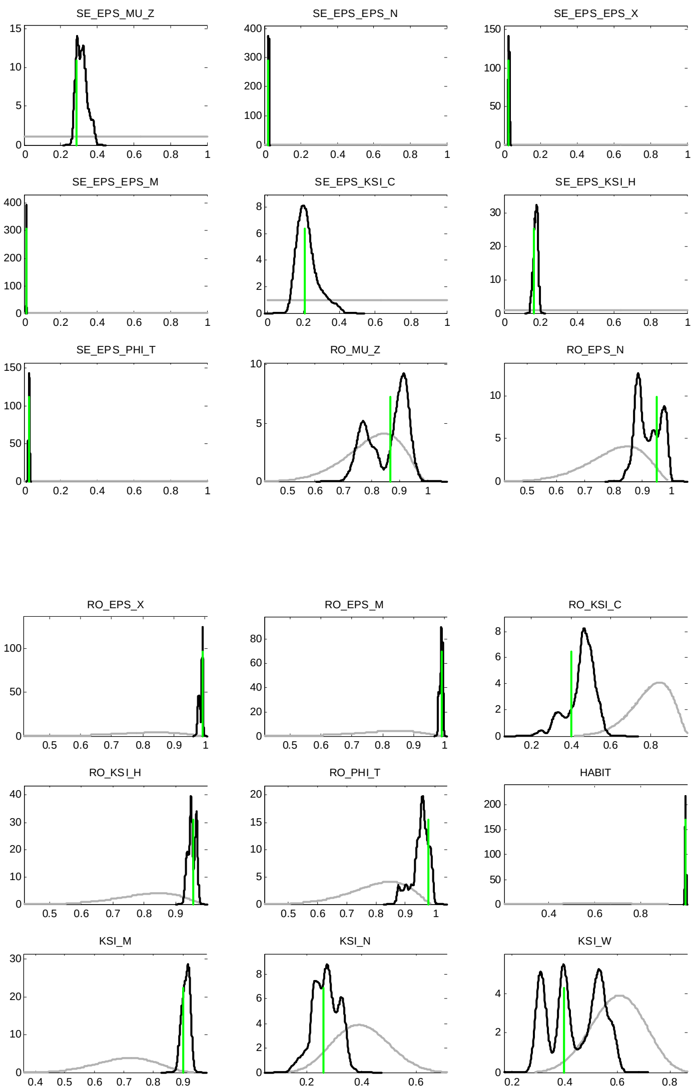

KPM
KPN
KPW
MU_S
GAMMA
OMEGA_INFL_C
OMEGA_S
OMEGA_Y

Table 2: Estimation results of Hungary

Param.	prior	prior mean	prior std	post. mode	post. mean	HDR (90%)	
ρ_{μ_z}	beta	0.8	0.1	0.8666	0.8569	0.7501	0.9468
ρ_{ϵ^N}	beta	0.8	0.1	0.9475	0.9202	0.8688	0.9870
ρ_{ϵ^X}	beta	0.8	0.1	0.9922	0.9863	0.9766	0.9940
ρ_{ϵ^m}	beta	0.8	0.1	0.9924	0.9885	0.9797	0.9958
ρ_{ξ^c}	beta	0.8	0.1	0.4016	0.4521	0.3304	0.5544
ρ_{ξ^H}	beta	0.8	0.1	0.9560	0.9511	0.9317	0.9691
ρ_{φ}	beta	0.8	0.1	0.9770	0.9498	0.8975	0.9906
b	beta	0.6	0.1	0.9931	0.9928	0.9900	0.9959
ξ^m	beta	0.7	0.1	0.8993	0.9094	0.8876	0.9299
ξ^N	beta	0.4	0.1	0.2618	0.2684	0.2050	0.3495
ξ^w	beta	0.6	0.1	0.3972	0.4373	0.2882	0.5683
κ^m	beta	0.4	0.1	0.1538	0.1393	0.0942	0.1829
κ^N	beta	0.4	0.1	0.3567	0.3081	0.1959	0.4548
κ^w	beta	0.4	0.1	0.3881	0.3404	0.2353	0.4472
μ_s	normal	5.0	1.5	10.9445	11.7449	9.9125	13.8239
γ	beta	0.2	0.1	0.1054	0.1093	0.0895	0.1319
ω_{π_c}	normal	1.78	0.3	1.6282	1.6262	1.6165	1.6358
ω_S	normal	0.15	0.05	0.0728	0.0709	0.0633	0.0762
ω_y	normal	0.6	0.2	0.0125	0.0122	0.0118	0.0125
$\sigma_{\varepsilon\mu_z}$	uniform	0.5	0.2887	0.2858	0.3154	0.2677	0.3604
σ_{ε^N}	uniform	0.5	0.2887	0.0222	0.0225	0.0204	0.0244
σ_{ε^X}	uniform	0.5	0.2887	0.0238	0.0256	0.0198	0.0306
σ_{ε^m}	uniform	0.5	0.2887	0.0109	0.0113	0.0097	0.0130
σ_{ε^c}	uniform	0.5	0.2887	0.2115	0.2236	0.1318	0.3189
σ_{ε^h}	uniform	0.5	0.2887	0.1620	0.1720	0.1540	0.1919
σ_{φ}	uniform	0.5	0.2887	0.0243	0.0264	0.0221	0.0322

Prior and posterior distributions of Hungary parameters



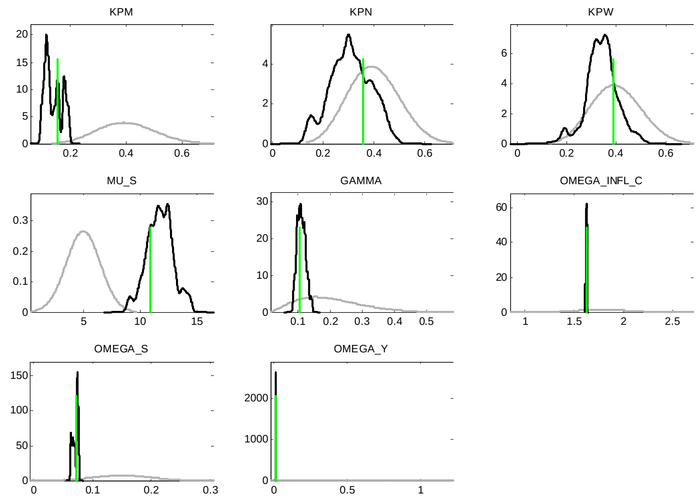
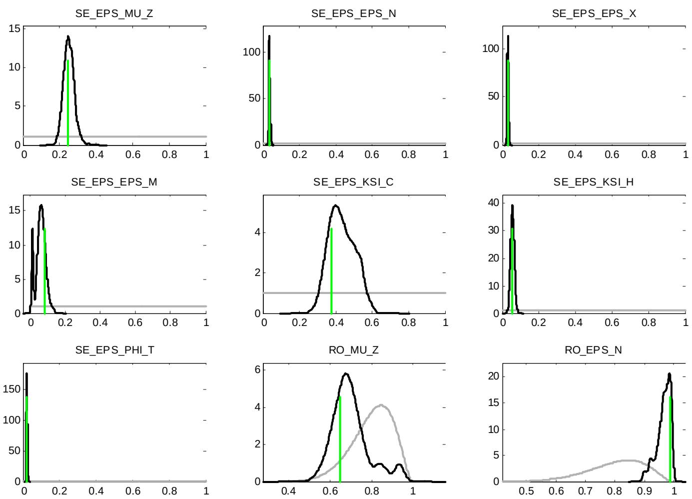
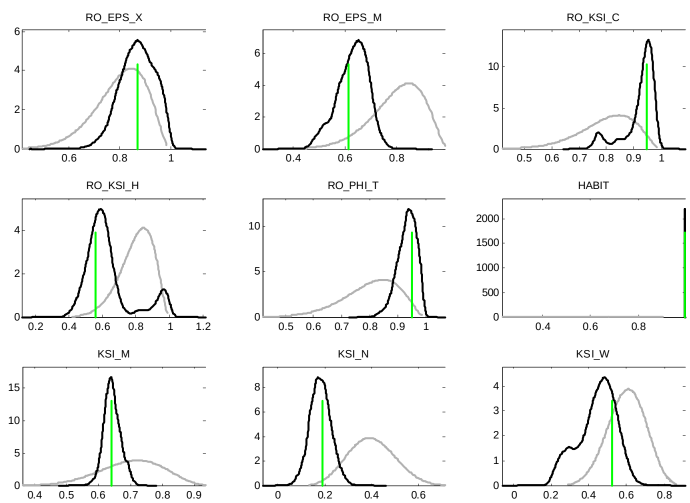

KPM
KPN
KPW
MU_S
GAMMA
OMEGA_INFL_C
OMEGA_S
OMEGA_Y

Table 3: Estimation results of Poland

Param.	prior	prior mean	prior std	post. mode	post. mean	HDR (90%)	
ρ_{μ_z}	beta	0.8	0.1	0.6513	0.6992	0.5577	0.8580
ρ_{ϵ^N}	beta	0.8	0.1	0.9874	0.9650	0.9273	0.9980
ρ_{ϵ^X}	beta	0.8	0.1	0.8693	0.8686	0.7713	0.9828
ρ_{ϵ^m}	beta	0.8	0.1	0.6120	0.6275	0.5279	0.7246
ρ_{ξ^C}	beta	0.8	0.1	0.9484	0.9229	0.8307	0.9925
ρ_{ξ^H}	beta	0.8	0.1	0.5604	0.6309	0.4814	0.9696
ρ_{φ}	beta	0.8	0.1	0.9491	0.9320	0.8809	0.9855
b	beta	0.6	0.1	0.9933	0.9930	0.9926	0.9933
ξ^m	beta	0.7	0.1	0.6397	0.6424	0.6017	0.6885
ξ^N	beta	0.4	0.1	0.1917	0.1819	0.1090	0.2562
ξ^w	beta	0.6	0.1	0.5235	0.4429	0.2454	0.5792
κ^m	beta	0.4	0.1	0.3276	0.3643	0.2005	0.5289
κ^N	beta	0.4	0.1	0.3721	0.3656	0.1943	0.5219
κ^w	beta	0.4	0.1	0.3342	0.3643	0.2080	0.5177
μ_s	normal	5.0	1.5	4.7641	5.9462	3.4516	8.4116
γ	beta	0.2	0.1	0.1790	0.2944	0.0882	0.4801
ω_{π_c}	normal	1.78	0.3	2.0266	1.7766	0.9543	2.2271
ω_S	normal	0.15	0.05	0.1608	0.1625	0.0836	0.2428
ω_y	normal	0.6	0.2	0.5279	0.4010	-0.0373	0.6134
$\sigma_{\varepsilon\mu_z}$	uniform	0.5	0.2887	0.2421	0.2501	0.2019	0.2952
σ_{ε^N}	uniform	0.5	0.2887	0.0334	0.0343	0.0284	0.0409
σ_{ε^X}	uniform	0.5	0.2887	0.0288	0.0303	0.0245	0.0359
σ_{ε^m}	uniform	0.5	0.2887	0.0796	0.0582	0.0286	0.0926
σ_{ε^C}	uniform	0.5	0.2887	0.3756	0.4320	0.3231	0.5484
σ_{ε^H}	uniform	0.5	0.2887	0.0510	0.0547	0.0384	0.0709
σ_{φ}	uniform	0.5	0.2887	0.0192	0.0209	0.0168	0.0249

Prior and posterior distributions of Poland's parameters





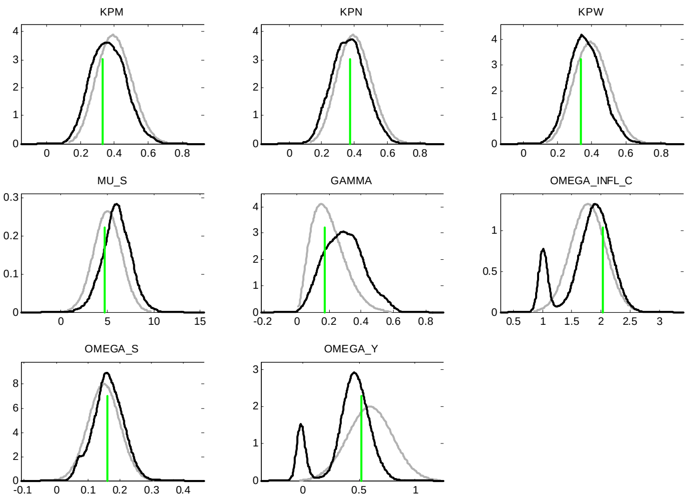
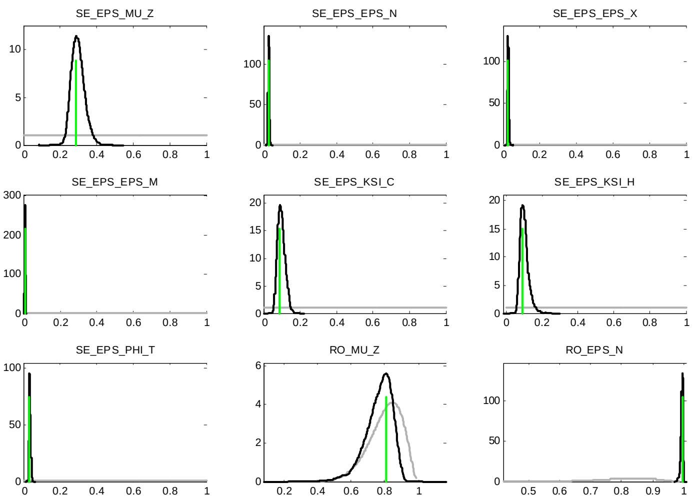
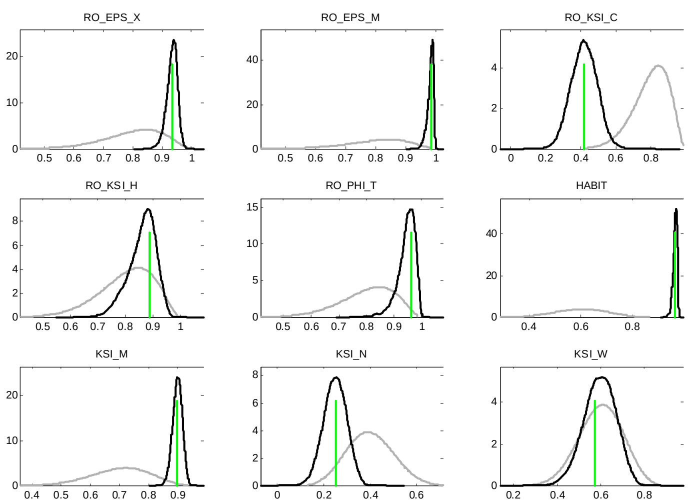

KPM
KPN
KPW
MU_S
GAMMA
OMEGA_INFL_C
OMEGA_S
OMEGA_Y

Table 1: Estimation results of Slovakia

Param.	prior	prior mean	prior std	post. mode	post. mean	HDR (90%)	
ρ_{μ_z}	beta	0.8	0.1	0.8066	0.7613	0.6419	0.8873
ρ_{ϵ^N}	beta	0.8	0.1	0.9961	0.9891	0.9891	0.9933
ρ_{ϵ^X}	beta	0.8	0.1	0.9349	0.9339	0.9044	0.9625
ρ_{ϵ^m}	beta	0.8	0.1	0.9867	0.9828	0.9676	0.9974
ρ_{ξ^c}	beta	0.8	0.1	0.4195	0.4185	0.2967	0.5369
ρ_{ξ^H}	beta	0.8	0.1	0.8867	0.8595	0.7766	0.9355
ρ_{φ}	beta	0.8	0.1	0.9617	0.9446	0.8994	0.9916
b	beta	0.6	0.1	0.9665	0.9672	0.9563	0.9800
ξ^m	beta	0.7	0.1	0.8972	0.8987	0.8723	0.9253
ξ^N	beta	0.4	0.1	0.2522	0.2484	0.2484	0.3298
ξ^w	beta	0.6	0.1	0.5722	0.5941	0.4788	0.7153
κ^m	beta	0.4	0.1	0.1908	0.1771	0.0941	0.2613
κ^N	beta	0.4	0.1	0.3701	0.3850	0.2184	0.5422
κ^w	beta	0.4	0.1	0.3961	0.3983	0.2379	0.5676
μ_s	normal	5.0	1.5	6.4664	6.2715	3.8473	8.6618
γ	beta	0.2	0.1	0.2569	0.2711	0.1231	0.4160
ω_{π_c}	normal	1.78	0.3	1.5486	1.6154	1.2996	1.9287
ω_S	normal	0.15	0.05	0.0580	0.0667	0.0124	0.1170
ω_y	normal	0.6	0.2	0.0241	0.0313	0.0059	0.0569
$\sigma_{\varepsilon\mu_z}$	uniform	0.5	0.2887	0.2836	0.2969	0.2396	0.3531
σ_{ε^N}	uniform	0.5	0.2887	0.0258	0.0269	0.0218	0.0319
σ_{ε^X}	uniform	0.5	0.2887	0.0256	0.0268	0.0215	0.0318
σ_{ε^m}	uniform	0.5	0.2887	0.0046	0.0053	0.0028	0.0075
σ_{ε^C}	uniform	0.5	0.2887	0.0861	0.0960	0.0625	0.1284
σ_{ε^H}	uniform	0.5	0.2887	0.0875	0.0996	0.0636	0.1355
σ_{φ}	uniform	0.5	0.2887	0.0318	0.0335	0.0262	0.0402

Prior and posterior distributions of Slovakia's parameters





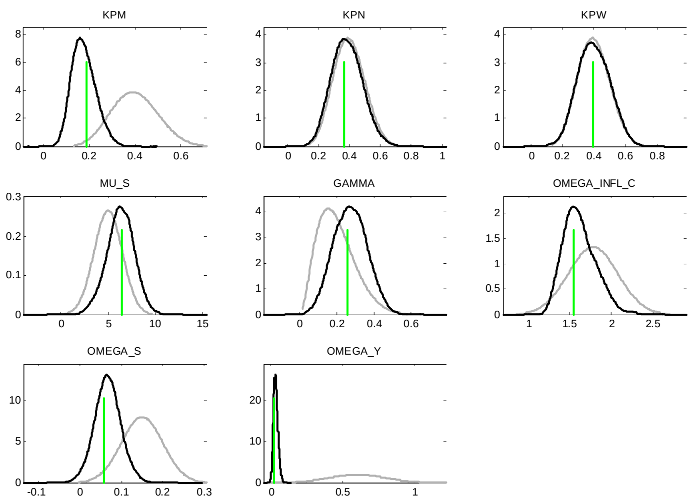
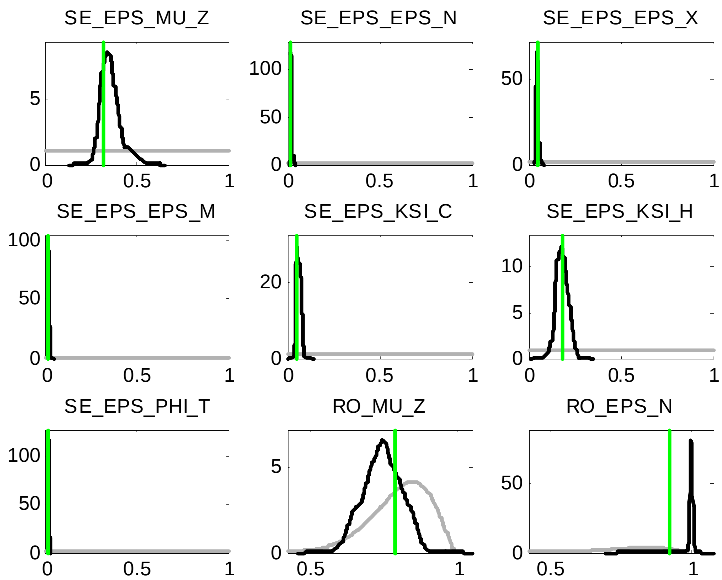
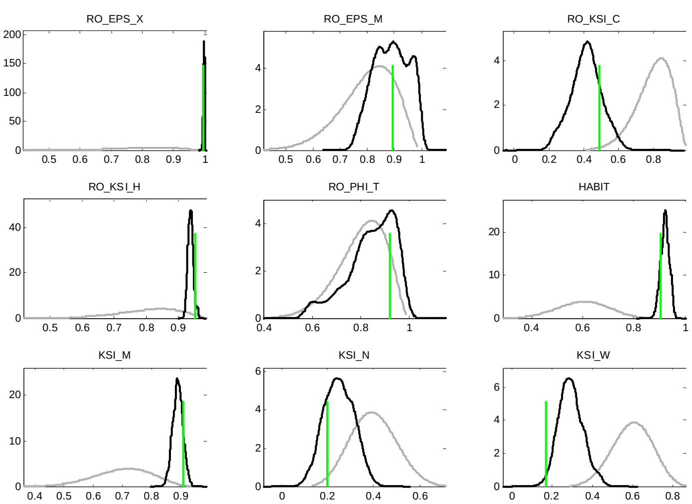

KPM
KPN
KPW
MU_S
GAMMA
OMEGA_INFL_C
OMEGA_S
OMEGA_Y

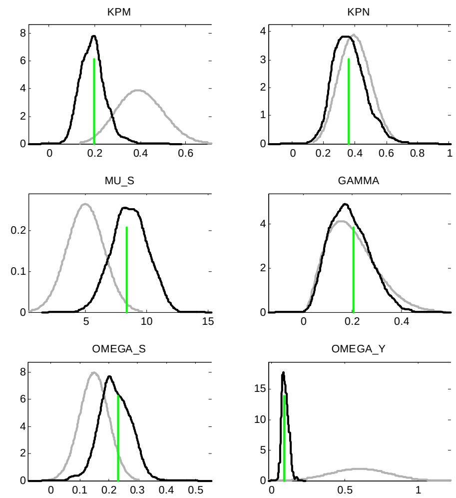
Table 1: Estimation results of Slovenia

Param.	prior	prior mean	prior std	post. mode	post. mean	HDR (90%)	
ρ_{μ_z}	beta	0.8	0.1	0.7879	0.7420	0.6344	0.8873
ρ_{ϵ^N}	beta	0.8	0.1	0.9214	0.9931	0.9931	0.9999
ρ_{ϵ^X}	beta	0.8	0.1	0.9940	0.9970	0.9942	0.9999
ρ_{ϵ^m}	beta	0.8	0.1	0.8922	0.8878	0.8021	0.9928
ρ_{ξ^c}	beta	0.8	0.1	0.4875	0.4124	0.2771	0.5764
ρ_{ξ^H}	beta	0.8	0.1	0.9507	0.9369	0.9225	0.9491
ρ_{φ}	beta	0.8	0.1	0.9210	0.8416	0.6947	0.9831
b	beta	0.6	0.1	0.9053	0.9226	0.8981	0.9525
ξ^m	beta	0.7	0.1	0.9086	0.8888	0.8617	0.9178
ξ^N	beta	0.4	0.1	0.1997	0.2484	0.1431	0.3497
ξ^w	beta	0.6	0.1	0.1681	0.2929	0.1892	0.3875
κ^m	beta	0.4	0.1	0.1973	0.1902	0.1074	0.2756
κ^N	beta	0.4	0.1	0.3639	0.3645	0.2047	0.5303
κ^w	beta	0.4	0.1	0.3871	0.4165	0.2484	0.5899
μ_s	normal	5.0	1.5	8.3468	8.6041	6.3180	11.0922
γ	beta	0.2	0.1	0.2032	0.1877	0.0582	0.3122
ω_{π_c}	normal	1.78	0.3	1.8308	2.1158	1.6789	2.4802
ω_S	normal	0.15	0.05	0.2325	0.2214	0.1370	0.3056
ω_y	normal	0.6	0.2	0.0862	0.0953	0.0606	0.1310
$\sigma_{\varepsilon\mu_z}$	uniform	0.5	0.2887	0.3159	0.3571	0.2757	0.4457
σ_{ε^N}	uniform	0.5	0.2887	0.0222	0.0248	0.0183	0.0294
σ_{ε^X}	uniform	0.5	0.2887	0.0503	0.0481	0.0374	0.0578
σ_{ε^m}	uniform	0.5	0.2887	0.0110	0.0145	0.0081	0.0214
σ_{ε^C}	uniform	0.5	0.2887	0.0516	0.0642	0.0458	0.0835
σ_{ε^H}	uniform	0.5	0.2887	0.1892	0.1835	0.1378	0.2406
σ_{φ}	uniform	0.5	0.2887	0.0151	0.0178	0.0127	0.0224

Prior and posterior distributions of Slovenia's parameters



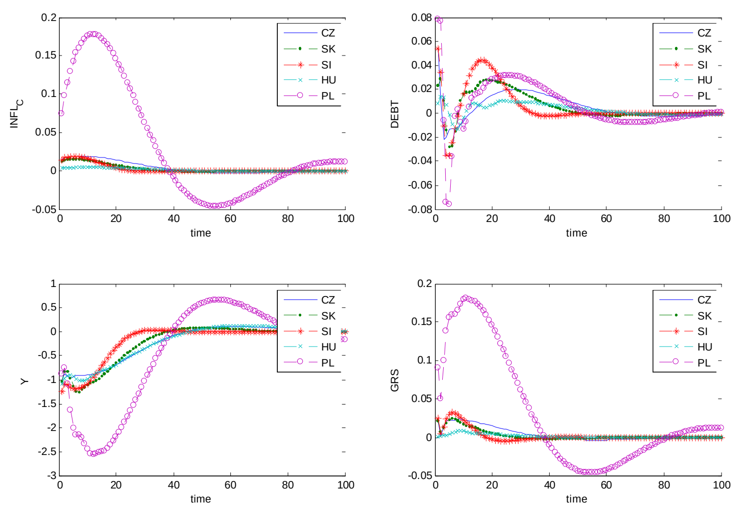




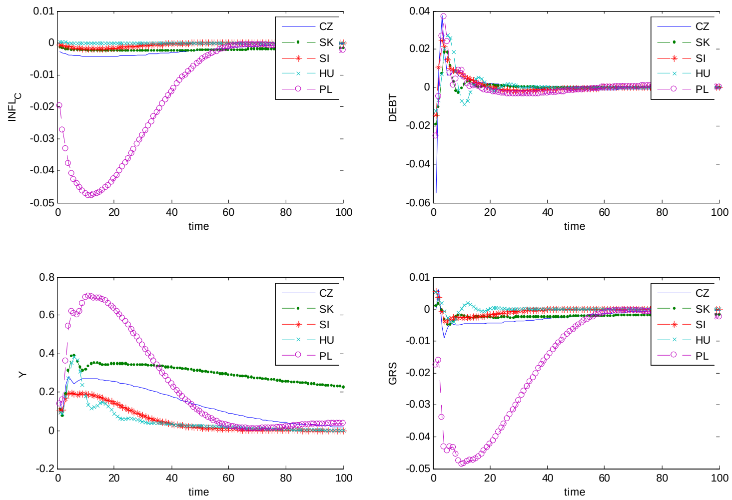
8.3 Optimal monetary policy

8.3.1 Impulse response functions of estimated shocks

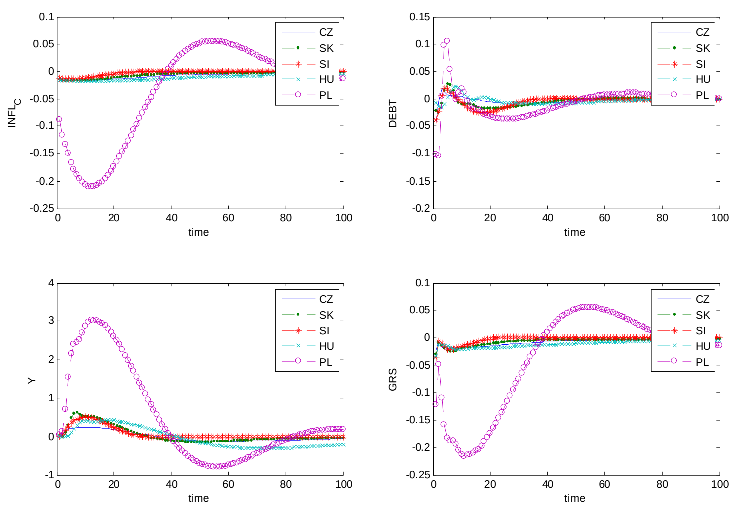
IRF of the unit-root productivity shock



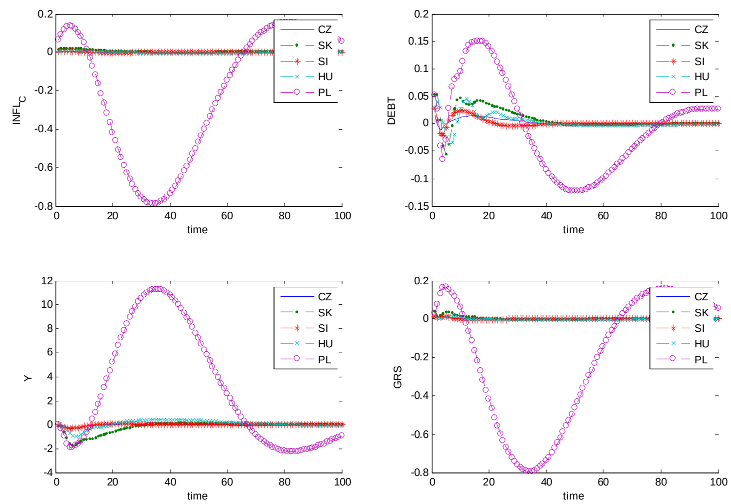
IRF of the stationary productivity shock



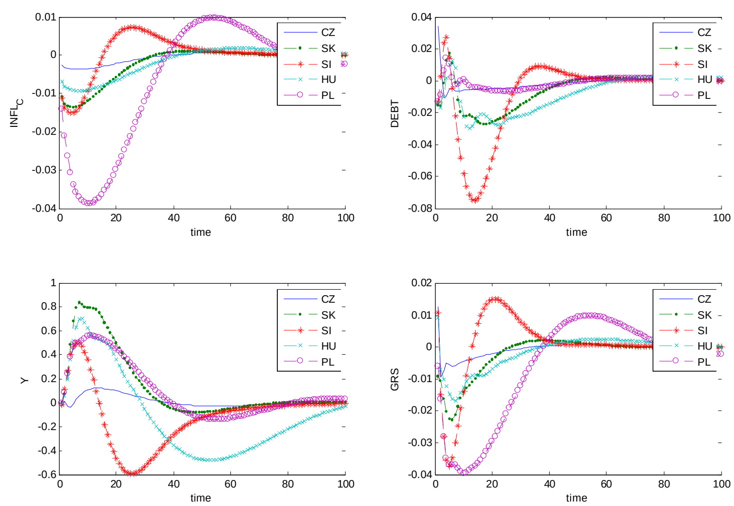
IRF of the monetary shock



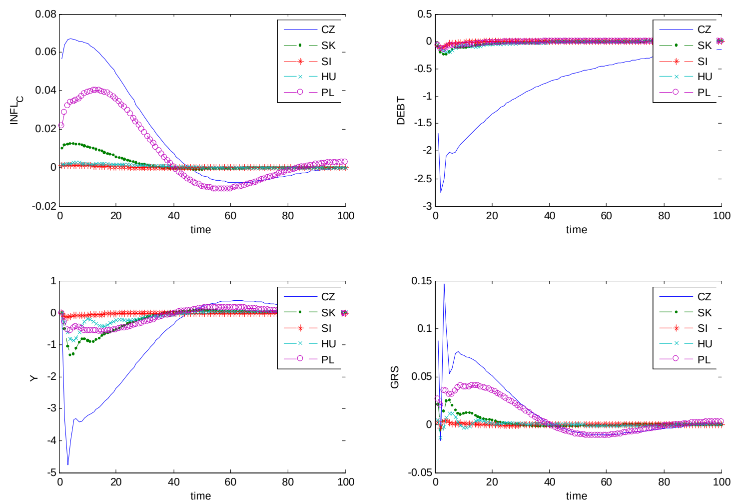
IRF of the consumption's preference shock



IRF of the labor's disutility shock

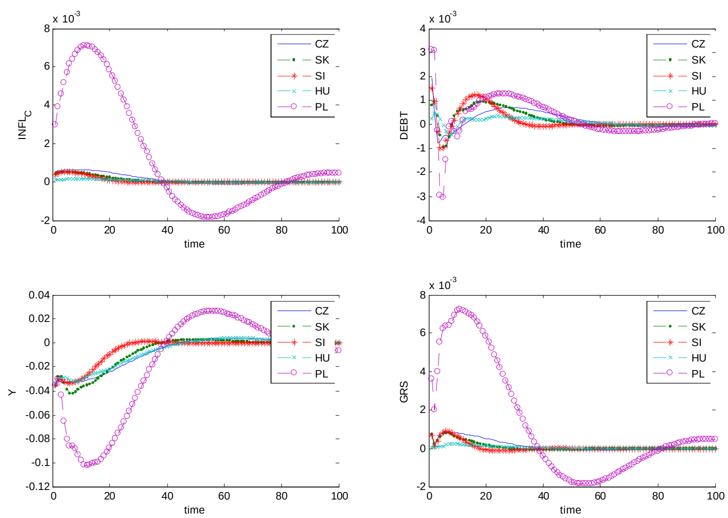


IRF of the risk premium shock

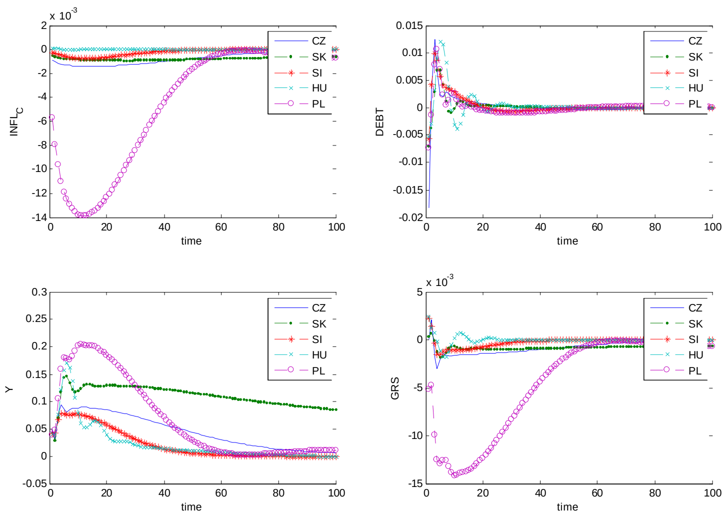


8.3.2 Impulse response functions of 1% shocks

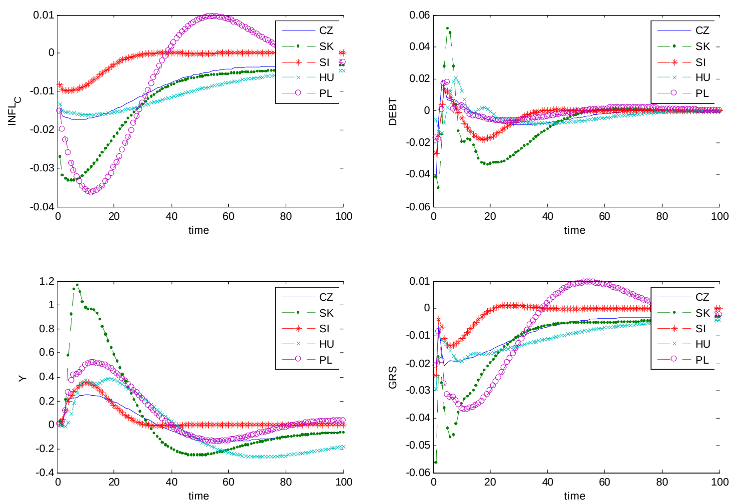
IRF of the unit-root productivity shock



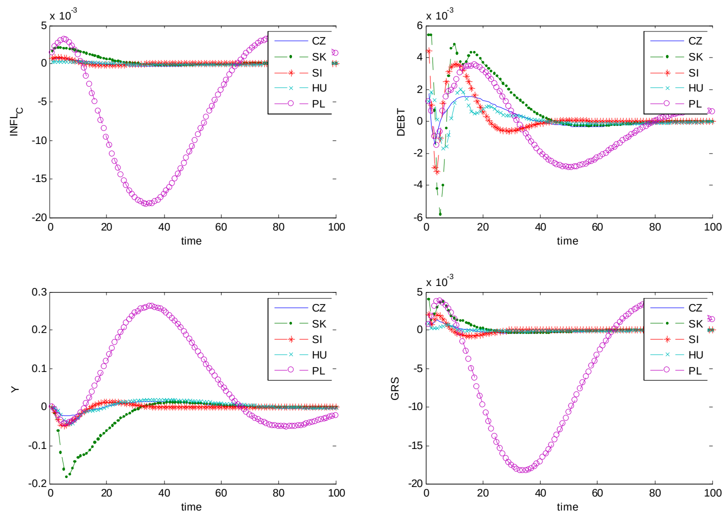
IRF of the stationary productivity shock



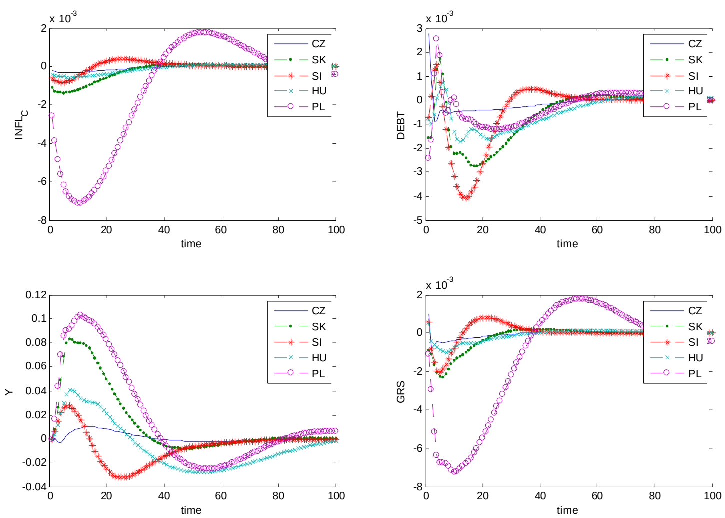
IRF of the monetary shock



IRF of the consumption's preference shock



IRF of the labor's disutility shock



IRF of the risk premium shock